Wavelet Frame Based Surface Reconstruction from Unorganized Points

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Abstract

Applications of wavelet frames to image restoration problems (e.g. image deblurring and inpainting) have been successful due to their redundancy and capability of sparsely approximating piecewise smooth functions like images (see e.g. [13, 14, 8]). However, wavelet frames have not yet been used for surface reconstruction problems. Recently in [6], connections between one of the wavelet frame based image restoration model [8, 28, 48] and variational models (e.g. the ROF model [45]) were rigorously established. Such connections not only grant new insights to wavelet frame based image restorations, it also case a geometric explanation to wavelet frame based approaches. This leads us to a wavelet frame based model, as well as a fast algorithm, to reconstruct implicit surfaces from unorganized point sets in $\mathbb{R}^3$. We will demonstrate the effectiveness of the proposed model using several commonly used examples.

Keywords: Surface reconstruction, variational method, total variation, wavelet frames, tight wavelet frames, split Bregman.

1. Introduction

Surface reconstruction from unorganized/scattered point sets (point clouds) is an important problem in geometric modelling. Given a set of scattered

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points $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\} \subset \mathbb{R}^3$ that are sampled from some unknown surface $\mathcal{S}$, the surface reconstruction problem is to construct a surface $\widehat{\mathcal{S}}$ from the observed data $\boldsymbol{X}$ such that $\widehat{\mathcal{S}}$ approximates $\mathcal{S}$. It has received a lot of attention in the computer graphics community in recent years because of the fast development of laser scanner technology which enables the point-based representation for highly detailed surfaces, and its wide applications in areas such as reverse engineering, product design, medical appliance design, archeology, etc. In this paper, we shall propose a wavelet frame based model to reconstruct surfaces from general point sets in $\mathbb{R}^3$.

1.1. Frames and Wavelet Frames

The theory of the multiresolution analysis (MRA) based frames, especially the MRA based tight wavelet frames, were extensively studied in the past two decades (see e.g. [20, 44, 21, 47, 25]). Examples of tight frames includes translation invariant wavelets [17], curvelets [9], and framelets [44], etc. In contrast to orthogonal bases, tight frames provide redundant representations to signals and images. The redundancy of tight frames usually leads to sparse approximation of piecewise smooth functions, like images, due to their short supports and high order of vanishing moments. Such property is known to be desirable for image restoration problems, like denoising, inpainting, deblurring, tomography, etc. This motivates the research on frame based image restorations [17, 9, 8, 16, 22, 28, 30, 11, 37].

Although wavelet frames are now widely used and quite successful in image restorations, there is not much work done for surface reconstruction problems. When the given set of points are sampled from the graph of a certain 2-dimensional function, a frame based model was proposed by [36], which used a simple principal shift invariant space and its associated wavelet transform to fit the given data points. However, for point sets that are sampled from general surfaces, there is not a reconstruction model in literature that is based on wavelet frames.

The major difficulty of using wavelet frames for general surface reconstruction problems is that the surface that needs to be recovered is not generally the graph of a certain scalar function. Therefore, we need to associate a function to a given surface in some way. One of the widely used techniques is to define the reconstructed surface $\widehat{\mathcal{S}}$ implicitly as the zero level set of a 3-dimensional scalar function u, i.e. $\widehat{\mathcal{S}} := \{x; u(x) = 0\}$. Then through minimizing some differential operator based variational model (see

e.g. [56, 53, 33, 52]), one can reconstruct u whose zero level is an approximations to the unknown surface. However, it is still not clear how one can apply wavelet frames to reconstruct such 3-dimensional scalar function u, until the recent work by [6]. In [6], the authors established a rigorous connection between differential operator based variational models (e.g. the ROF model [45]) and one of the frame based model, called analysis based approach [8, 28, 48]. Their discovery shows that when spline wavelet frames of [44] are used, the analysis based approach can be viewed as certain finite difference approximation to various variational models at a given resolution. Based on the connections found by [6], we can combine the ideas of variational models with those of frame based models. Such combination was already used by [24] where a wavelet frame based image segmentation model was proposed.

Motivated by the image segmentation model of [24] and the discoveries of [6], and inspired by the variational models [56, 53, 33, 52], we shall propose a wavelet frame based model for surface reconstruction problems. With this new model, the efficient algorithms for frame based image restoration models can now be used to obtain fast algorithms for surface reconstruction.

1.2. Brief Literature Review

There has been tremendous work done for surface reconstruction from point clouds. One of the earliest algorithm was given by [35] where the authors locally estimated the signed distance function of the true surface. Smooth surfaces can also be built by fitting radial basis functions to a given point cloud [50], which was later adapted to large data sets by [10]. In [56], a variational level set method was proposed, where the authors introduced an energy functional utilizing the distance function associated to the given data points and minimized the energy by solving a level set equation. Recently, some work that extends the work by [56] were proposed [53, 33, 52].

Instead of computing a single global approximation to point clouds, the approaches of using moving least squares algorithms locally fit smooth functions to each sample point and then smoothly combine them together [41, 38]. A different approach to moving least squares is the nonlinear projection method originally proposed by [39]. A point-set surface is defined as the set of stationary points of a projection operator, which was first used by [1] for point based modelling and rendering. Another popular technique is to construct a polyhedral surface from the input set of points utilizing the Voronoi diagram [2, 4, 3, 40, 23], which is closely related to Voronoi-based algorithms for medial axis estimation.

1.3. Plan of The Paper

In Section 2, we will first present a general variational model for surface reconstruction problems that includes some of the known models as examples. Then, after a brief introduction to (tight) wavelet frames, we will propose a wavelet frame based model that can be regarded as certain discretizations to the general variational model. To show the advantage of using wavelet frames over the standard finite difference discretization for differential operators, we will also present a 2-dimensional comparison in this section. In Section 3, we will present a fast algorithm that solves the frame based model using the idea of split Bregman algorithm. Numerical results for surface reconstruction will be given at the end of this section.

2. Mathematical Models

Given a set of scattered points $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\} \subset \mathbb{R}^3$ that are sampled from some unknown surface $\mathcal{S}$, we need to find an approximation to $\mathcal{S}$ based on the observed data $\boldsymbol{X}$. If we restrict our interests to surfaces that can be implicitly represented by the zero level set of some function $u : \Omega \mapsto \mathbb{R}$, where $\Omega \subset \mathbb{R}^3$ is some computation domain, then the surface reconstruction problem is amount to finding u such that $\{x \in \mathbb{R}^3 : u(x) = 0\}$ interpolates or approximates the data $\boldsymbol{X}$. Many variational models were proposed in the literature to recover a desirable level set function u for the given data $\boldsymbol{X}$. However, there has not been any work on applying wavelet frames, which has been widely used for image restorations, for surface reconstruction. One of the reasons is the lack of geometric interpretations of wavelet frames. Recently, the authors of [6] established a link between one of frame based image restoration models with variational models, which, for the very first time, grants geometric interpretations to wavelet frames. Based on the work of [6], we will propose a wavelet frame based model for surface reconstruction problems.

We will start this section by presenting a general variational model for surface reconstructions which includes some of the known model in the literature as special examples. Then, after a brief introduction of wavelet frames, we will present the wavelet frame based formulation for surface reconstructions which can be regarded as certain discretization of the general variational model. However, once it links to the wavelet frame, the advantages of using wavelet frame immediately show up. First, there are fast algorithms

for frame based optimizations that can also be used for surface reconstruction. Secondly, the wavelet tight frame provides a rich family of difference operators that can be understood as various discretizations of a family of differential operators. The multi-level nature of the tight wavelet frame decomposition automatically applies different difference operators adaptively to the geometric structure of the surface, hence, features of the surface can be well reconstructed. This also leads to the fact that wavelet frame based approaches outperform those by discretizing variational models without using wavelet frame structures.

2.1. Variational Models for Surface Reconstruction

Like the variational models proposed for image restoration problems, we can define a general variational model for surface reconstruction as

$$\min_{u \in V} \ \|\boldsymbol{\nu} \cdot \boldsymbol{D}(u)\|_{1,p} + H(u, f), \tag{2.1}$$

where V is some convex set, $\boldsymbol{D} := \{D_{\boldsymbol{j}} : 1 \leq |\boldsymbol{j}| \leq s\}$ is a vector of differential operators of order s, $H(u, f)$ is some smooth convex functional and f is some given function that may be obtained from the input point set. The norm $\|\cdot\|_{1,p}$ is defined as follows

$$\|\boldsymbol{\nu} \cdot \boldsymbol{D}(u)\|_{1,p} := \left\| \left(\sum_{1 \leq |\boldsymbol{j}| \leq s} \nu_{\boldsymbol{j}} |D_{\boldsymbol{j}} u|^p \right)^{\frac{1}{p}} \right\|_1, \tag{2.2}$$

where $\nu_{\boldsymbol{j}}$ are some pre-selected weight functions defined on Ω, and $\|\cdot\|_q$ denotes the L_q-norm. Among all the different choices of $\nu_{\boldsymbol{j}}$, the choice of distance function is very popular and was shown to be effective in the literature. For a given set of scattered points $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\} \subset \mathbb{R}^3$, define the distance function as

$$\varphi(x) := \inf_{y \in \boldsymbol{X}} \|x - y\|_2, \quad x \in \Omega, \tag{2.3}$$

which can be obtained by solving the Eikonal equation [19, 55] (see Figure 1 for an example of $\varphi(x)$). Then we can choose, e.g. $\nu_{\boldsymbol{j}} = \alpha\varphi$, for each $1 \leq |\boldsymbol{j}| \leq s$ and some scalar $\alpha \in \mathbb{R}$. The distance function was first used in [56] for surface reconstruction problems, and was used and analyzed by various later work (see e.g. [43, 27, 52]).

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The general model (2.1) includes some of the known models in the literature as special cases. For example when $V = L_2(\Omega)$, $\boldsymbol{D} = \nabla$ and $H(u, f) = \frac{1}{2h}\|u - f\|_2^2$, model (2.1) was used iterative in [33] for surface reconstructions. Another example is when $V = \{u \in L_2(\Omega) : 0 \le u \le 1\}$, $\boldsymbol{D} = \nabla$ and $H(u, f) = \langle 2f-1, u\rangle$, model (2.1) was used in [52] as the first step of their entire procedure, which is also a special case of the Chan-Vese model [12, 15] (called active contour with edges) proposed for image segmentation problems.

The variational model (2.1) views the variable u, as well as f and φ as functions defined on Ω. However, the data $\boldsymbol{X}$ collected by certain scanning machine is discrete and the distance function φ that one can compute is also discrete. Therefore, when we are solving the problem (2.1), we need proper discretizations of it. Instead of using standard discretizations for differential operators, we shall use wavelet frames which was proven in [6] to be a consistent approximation when meshsize goes to zero. Furthermore, the discretization provided by wavelet frames was shown, in e.g. [13, 14, 7, 8, 6, 25], to be superior than the standard discretization for some of the variational models (e.g. total variation based models) for image restoration problems, and also for image segmentation problems [24]. We shall leave the details to Section 2.3 after a brief introduction to (tight) wavelet frames in Section 2.2.

2.2. Tight Wavelet Frames

We now briefly introduce the concept of tight frames and framelets. Interesting readers should consult [20, 44, 21] for theories of frames and framelets, [47] for a short survey on theory and applications of frames, and [25] for a more detailed survey.

A countable set $X \subset L_2(\mathbb{R})$ is called a tight frame of $L_2(\mathbb{R})$ if

$$f = \sum_{h \in X} \langle f, h\rangle h \quad \forall f \in L_2(\mathbb{R}),$$

where $\langle \cdot, \cdot\rangle$ is the inner product of $L_2(\mathbb{R})$. For given $\Psi := \{\psi_1, \ldots, \psi_r\} \subset L_2(\mathbb{R})$, the wavelet system generated by Ψ is defined by the collection of the dilations and the shifts of Ψ as

$$X(\Psi) := \{\psi_{\ell,j,k} : 1 \le \ell \le r; \; j, k \in \mathbb{Z}\} \quad \text{with} \quad \psi_{\ell,j,k} := 2^{j/2}\psi_\ell(2^j \cdot -k).$$

When $X(\Psi)$ forms a tight frame of $L_2(\mathbb{R})$, it is called a tight wavelet frame, and ψ_ℓ, $\ell = 1, \ldots, r$, are called framelets.

The construction of framelets can be obtained by the unitary extension principle (UEP) of [44]. In our implementations, we will use the piecewise linear B-spline framelets constructed by [44]. Given a 1-dimensional framelet system for $L_2(\mathbb{R})$, the s-dimensional tight wavelet frame system for $L_2(\mathbb{R}^s)$ can be easily constructed by using tensor products of 1-dimensional framelets (see e.g. [20, 25]).

In the discrete setting, a discrete image u is an s-dimensional array. We will use W to denote fast tensor product framelet decomposition and use $W^\top$ to denote the fast reconstruction. Then by the unitary extension principle [44], we have $W^\top W = I$, i.e. $u = W^\top W u$ for any image u. We will further denote an L-level framelet decomposition of u as

$$Wu = \{W_{l,\boldsymbol{j}}u : 0 \leq l \leq L-1, \boldsymbol{j} \in I\},$$

where I denotes the index set of all framelet bands. More details on discrete algorithms of framelet transforms can be found in [25].

2.3. Wavelet Frame Based Surface Reconstruction Model

We now propose a wavelet frame based model for surface reconstruction problems, which serves as discretizations of the general variational model (2.1). Given data $\boldsymbol{X}$, we will still use φ to denote the distance function associated to $\boldsymbol{X}$. However, φ, so are all other variables involved in the model and the algorithm, should be understood as a discrete array defined on regular grids in $\mathbb{R}^3$.

Surface Reconstruction Model

Given data $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\} \subset \mathbb{R}^3$, solve the following optimization problem

$$\min_{u \in V} \|\boldsymbol{\lambda} \cdot Wu\|_{1,p} + H(u, f), \tag{2.4}$$

and

$$\|\boldsymbol{\lambda} \cdot Wu\|_{1,p} := \left\| \sum_{l=0}^{L-1} \left(\sum_{\boldsymbol{j} \in I} \lambda_{l,\boldsymbol{j}} |W_{l,\boldsymbol{j}}u|^p \right)^{\frac{1}{p}} \right\|_1. \tag{2.5}$$

The parameter $\boldsymbol{\lambda}$ is chosen as

$$\lambda_{l,\boldsymbol{j}} = \varphi(x)^q$$

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for each l and $\boldsymbol{j}$ with $0 < q \le 1$. In particular, we shall focus on the following choice of the set V and $H(u, f)$ due to their simplicity and effectiveness in practice:

$$V = \{0 \le u \le 1\} \quad \text{and} \quad H(u, f) = \mu \langle 2f - 1, u \rangle. \tag{2.6}$$

Remark 1.

1. *The norm $\|\cdot\|_{1,1}$ as given by (2.5) is the standard ℓ_1-norm used for frame based image restoration problems, while the norm $\|\cdot\|_{1,2}$ is an isotropic version of $\|\cdot\|_{1,1}$. It was shown by [6] that for image restorations, the $\|\cdot\|_{1,2}$ outperforms $\|\cdot\|_{1,1}$ in terms of both quality of the restoration and efficiency of the corresponding numerical algorithm. Therefore, in this paper, we shall use the norm $\|\cdot\|_{1,2}$.*

2. *When we choose V and $H(u, f)$ as in (2.6), we take f as a characteristic function, i.e. $f = \chi_\Lambda$, where $\partial \Lambda$ is an initial approximation to the the given data set $\boldsymbol{X}$. In other words, the role of f here is an initial guess of the underlying surface that we want to reconstruction.*

3. *The analysis in [6] suggests that under proper assumptions on the space V and $H(u, f)$ and proper choices of $\boldsymbol{\lambda}$, the energy functional of the frame based model (2.4) is indeed a discrete approximation to the energy functional of the variational model (2.4). In particular, if one assumes that u is smooth enough, such connection can be easily established via Taylor's expansions.*

For some of the variational models (e.g. total variation based models), using the discretization provided by wavelet frames has advantages over the standard discretization for image restoration problems (see e.g. [13, 14, 7, 8, 6, 25]), and also for image segmentation problems [24]. By putting model (2.1) into a wavelet frame setting, one can use a multiresolution structure to adaptively choose a proper differential operators in different regions of a given image according to the order of the singularity of underlying solutions. It should be pointed out here that if one wants to use a more general differential operator in model (2.1), the ability of applying different differential operators according to where various singularity are located is the key to make such generalization successful. The wavelet frame based approach has a built-in adaptive mechanism via the multiresolution analysis that provides a natural tool for this purpose. To illustrate the benefit of using the frame based model, we present a 2-dimensional comparison between model (2.4) and model (2.1)

(using a standard discretization) for the case $\boldsymbol{D} = \nabla$, and V and $H(u, f)$ given as in (2.6) (see Figure 1). Note that the curves shown in Figure 1 are the boundaries of the set $\{u^\star \geq 0.5\}$ where $u^\star$ is the solution to the corresponding optimization problems.
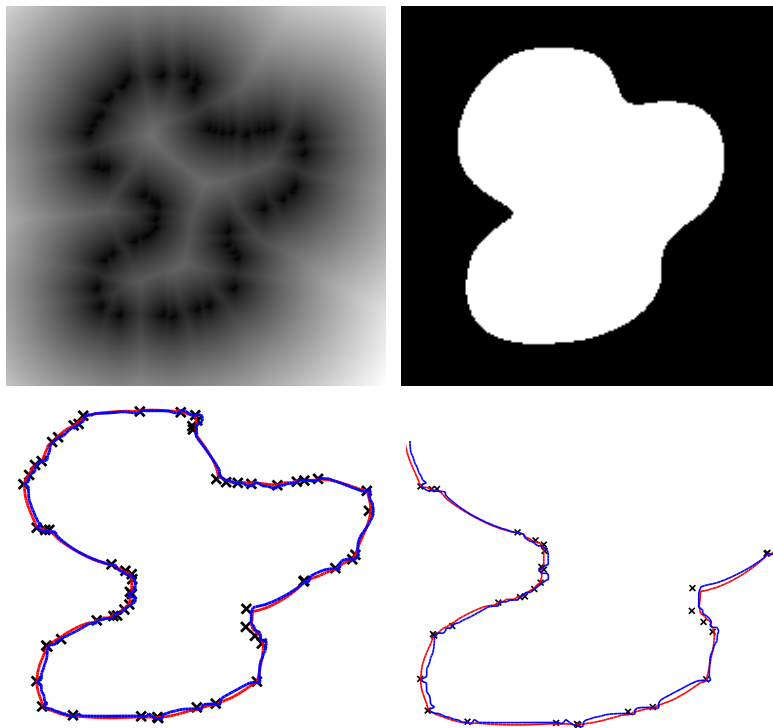

Figure 1: Upper left: distance function $\varphi(x)$ to a given point set; upper right: function f, the boundary of its support is an initial approximation of the given data points; lower left: reconstructed curves using tight wavelet frame (red) and total variation (blue); lower right: a zoom-in view of the lower left image.

3. Algorithm and Numerical Experiments

In this section, we focus on the algorithm and numerical results of the model (2.4) with V and $H(u, f)$ given as in (2.6), i.e. we will use the following model

$$\min_{0 \leq u \leq 1} \|\boldsymbol{\lambda} \cdot Wu\|_{1,p} + \mu\langle 2f - 1, u\rangle. \tag{3.1}$$

3.1. Fast Algorithm

We now describe an algorithm that solves (3.1). Note that the corresponding algorithm for the general frame based model (2.4) can be derived in a similarly way.

The proposed algorithm is based on the split Bregman algorithm. The split Bregman algorithm was first proposed in [34] which was shown to be powerful in [34, 54] when it is applied to various PDE based image restoration approaches, e.g., ROF model [45] and nonlocal variational models [31]. Convergence analysis of the split Bregman algorithm were given in [8] for frame based image restoration problems.

The idea of split Bregman algorithm is to first replace the term Wu in (3.1) by a new variable d and then adds a new constraint $d = Wu$ into (3.1). Then (3.1) is now equivalent to

$$\min_{0 \leq u \leq 1, d=Wu} \|\boldsymbol{\lambda} \cdot d\|_{1,p} + \mu \langle r, u \rangle, \tag{3.2}$$

where both u and d are variables need to be optimized. In order to solve (3.2), an iterative algorithm based on the Bregman distance [5, 42] with an inexact solver was proposed in [34]. This leads to the alternative split Bregman algorithm for (3.2). The derivation of splitting Bregman algorithm in [34, 8] is based on Bregman distance. It was recently shown (see e.g. [29, 49]) that the split Bregman algorithm can also be derived by applying augmented Lagrangian method (see e.g. [32]) on (3.2). The connection between split Bregman algorithm and Douglas Rachford splitting was addressed by [46]. We shall skip the detailed derivations and recall the split Bregman algorithm that solves (3.1) as follows

$$\begin{cases} u^{k+1} = \arg\min_{0 \leq u \leq 1} \mu \langle r, u \rangle + \frac{\nu}{2} \|Wu - d^k + b^k\|_2^2, \\ d^{k+1} = \arg\min_d \|\boldsymbol{\lambda} \cdot d\|_{1,p} + \frac{\nu}{2} \|d - Wu^{k+1} - b^k\|_2^2, \\ b^{k+1} = b^k + \delta(Wu^{k+1} - d^{k+1}). \end{cases} \tag{3.3}$$

We propose the following two steps to approximate the solution u^{k+1} of the first subproblem of (3.3):

$$\begin{cases} u^{k+\frac{1}{2}} &= W^\top(d^k - b^k) - \frac{\mu}{\nu} r, \\ u^{k+1} &= \min\{\max\{u^{k+\frac{1}{2}}, 0\}, 1\}. \end{cases} \tag{3.4}$$

The second subproblem of (3.3) can be solved rather efficiently by shrinkage (see e.g. [26, 18] for the case $p = 1$ and [51] for the case $p = 2$):

$$d^{k+1} = \mathcal{T}^p_{\boldsymbol{\lambda}/\nu}(Wu^{k+1} + b^k), \qquad (3.5)$$

where the thresholding operator $\mathcal{T}^p_\tau(v)$ is defined, for each $0 \leq l \leq L-1$ and $\boldsymbol{j} \in I$, as

$$\left(\mathcal{T}^p_\tau(v)\right)_{l,\boldsymbol{j}} := \begin{cases} \frac{v_{l,\boldsymbol{j}}}{|v_{l,\boldsymbol{j}}|} \max\{|v_{l,\boldsymbol{j}}| - \tau, 0\}, & p = 1 \\ \frac{v_{l,\boldsymbol{j}}}{R_l} \max\{R_l - \tau, 0\}, & p = 2 \end{cases}$$

where $R_l := \left(\sum_{\boldsymbol{j} \in I} |v_{l,\boldsymbol{j}}|^2\right)^{\frac{1}{2}}$.

Now combining (3.4) and (3.5), we obtain Algorithm 1 for the surface reconstruction model (3.1):

Algorithm 1 Wavelet Frame Based Surface Reconstruction

Given a binary image f and $r = 2f - 1$, initialize $d = b = 0$.
while topping criteria is not met **do**

    1. Update u:
$$\begin{cases} u & \leftarrow W^\top(d - b) - \frac{\mu}{\nu}r, \\ u & \leftarrow \min\{\max\{u, 0\}, 1\}. \end{cases}$$

    2. Update d:
$$d \leftarrow \mathcal{T}^p_{\boldsymbol{\lambda}/\nu}(Wu + b).$$

    3. Update b:
$$b \leftarrow b + \delta(Wu - d).$$

  end while

3.2. Numerical Experiments

We now present some numerical results for surface reconstruction using Algorithm 1. A summary of the data we are using is given by Table 1. The data "hand" is obtained from "www.cc.gatech.edu/projects/large models", while all the other data in Table 1 is obtained from "www-graphics.stanford.edu/data/3Dscanrep".

To get better initialization f, we adopt the idea proposed by [52]. Given a point set $\boldsymbol{X}$ and its corresponding distant function $\varphi(x)$, we compute f by

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solving the following Eikonal using fast sweeping method [55]

$$|\nabla f| = \frac{1}{\varphi^2 + \varepsilon},$$

where ε is some properly chosen parameter. We shall skip the details here, but interested reader should consult [52, 55] for details. For all our experiments, we shall choose $q = \frac{1}{2}$. The reason that we are in favor of $q = 0.5$ over $q = 1$ (used in [56]) is because when the true surface that needs to be reconstructed has rather concave regions, using $q = 0.5$ will require a less restrictive initialization than using $q = 1$.

All calculations are done in MATLAB on a laptop with Intel Core i7 (1.73 GHz) CPU and 8.0G RAM. The stopping criteria we choose for Algorithm 1 is

$$\frac{\|u^{k+1} - u^k\|_2}{\|u^k\|_2} < 5 \times 10^{-4}.$$

The total number of iterations and computational time for each data set are given in Table 1. Reconstructed surfaces, as well as their corresponding initializations f (the boundaries of the supports of f are visualized) are shown by Figure 2-6. All surfaces are visualized as the 0.5-level set of their corresponding level set functions.

Table 1: Data summary and computation efficiency of Algorithm 1.

Model	Data points	Grid size	#Iterations	Time (min.)
Armadillo	172,974	$164 \times 141 \times 130$	54	11.7
Budda	144,647	$277 \times 126 \times 126$	109	33.4
Bunny	35,947	$100 \times 100 \times 83$	57	3.6
Dragon	437,645	$162 \times 221 \times 110$	95	25.8
Hand	327,323	$161 \times 221 \times 90$	83	18.3

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Figure 2: Model: armadillo. Left image: initialization; right image: reconstruction.

Figure 3: Model: budda. Left image: initialization; right image: reconstruction.

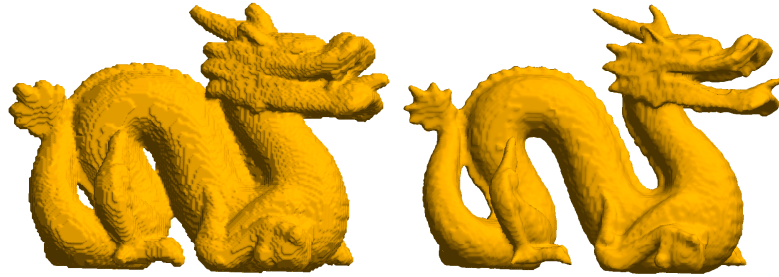Figure 4: Model: bunny. Left image: initialization; right image: reconstruction.

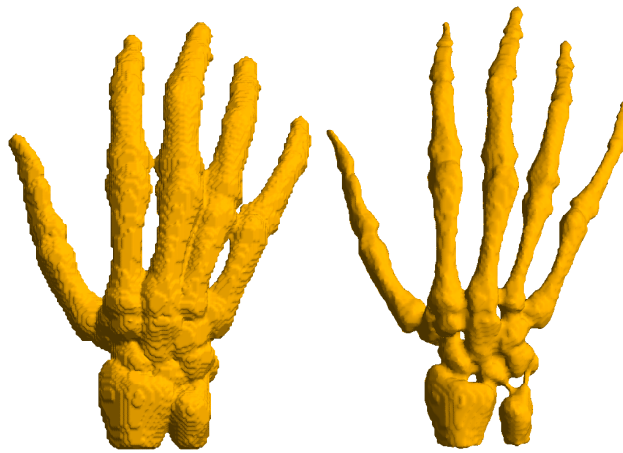Figure 5: Model: dragon. Left image: initialization; right image: reconstruction.

Figure 6: Model: hand. Left image: initialization; right image: reconstruction.

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