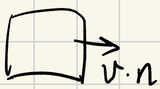
科学机器学习简介

1、可压缩NS方程

ρ：密度　$\vec{v}$：速度

① 质量守恒



$$m=\int \rho\cdot d\Omega$$

$$\Delta m = -\int \rho(v\cdot n)\cdot \Delta t\, dS$$

$$\frac{\Delta m}{\Delta t} = \frac{d}{dt}\int \rho\, d\Omega = -\int \rho\, v\cdot n\, dS$$

$$= -\int \mathrm{div}(\rho v)\, dS$$

$$\frac{d\rho}{dt} + \nabla(\rho\vec{v}) = 0$$

② 动量守恒

质量交互

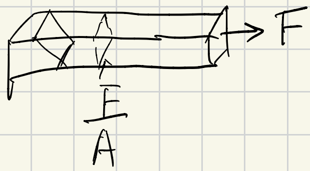
$$\Delta\int \rho\vec{v}\cdot d\Omega = -\int (\rho\vec{v})(v\cdot n)\,\Delta t\, dS$$

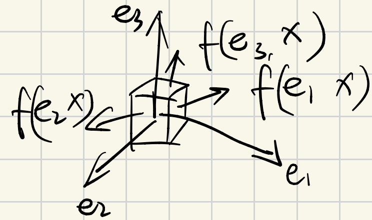
(pascal)

$$+\int f(n,x)\, dS \cdot \Delta t$$

表面作用力

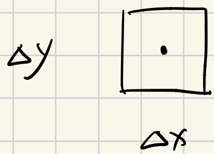
$f(n,x)$：应力，物体由于外因而变形时，在物体内各部分之间产生相互作用的内力，单位面积上的内力称为应力。





$$f(n, \vec{x}) = \bar{\bar{\sigma}} \cdot \vec{n} \qquad \bar{\bar{\sigma}} : 应力张量 (R^{3\times3})$$

σ : 对称 (2D 情况)，考虑小方块



$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$

角动量守恒: $\Sigma\, r_i \times f_i = I\dot{\omega} \qquad I = \int \rho |r|^2 dV \propto (\Delta x \Delta y)^2$

右边 : $\frac{\Delta x}{2} \vec{e}_x \times (\sigma \cdot \vec{e}_x + O(\Delta y))\Delta y = \frac{\Delta x \Delta y}{2} \sigma_{xy}$

上边 : $\frac{\Delta y}{2} \vec{e}_y \times (\sigma \cdot \vec{e}_y + O(\Delta x))\Delta x = -\frac{\Delta x \Delta y}{2} \sigma_{yx}$

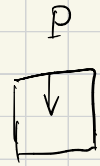
$$\Delta x \Delta y (\sigma_{xy} - \sigma_{yx}) = I\dot{\omega}$$

$$\Rightarrow \sigma_{xy} - \sigma_{yx} \approx O(\Delta x \Delta y)$$

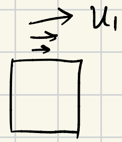
$$\Rightarrow \sigma_{xy} = \sigma_{yx}$$

流体：

应力 $\bar{\bar{\sigma}}\cdot\vec{n} = -P\vec{n} + \bar{\bar{\tau}}\cdot\vec{n}$



$$\bar{\bar{\tau}} = \bar{\bar{\tau}}(\nabla\vec{v})$$



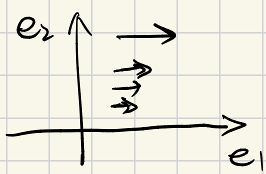
σ：对称（角动量守恒）

$$\bar{\bar{\tau}} = \bar{\bar{\tau}}(\nabla\vec{v})$$

牛顿流体（假设）

$$\bar{\bar{\tau}} = \mu(\nabla\vec{v} + \nabla\vec{v}^T) - \frac{2}{3}\mu(\nabla\cdot\vec{v})I$$

$$\begin{bmatrix} 2\mu\frac{\partial v_1}{\partial x_1} - \frac{2}{3}\mu\nabla\cdot\vec{v} & \mu\left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right) & \\ \mu\left(\frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2}\right) & \ddots & \\ & & \ddots \end{bmatrix}$$



$$n = (0, 1, 0)$$

$$f = \sigma n = \left[\ \mu\left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right) \cdots\cdots \right]$$

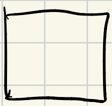
$$\frac{d}{dt}(\rho\vec{v}) + \begin{pmatrix} \nabla\cdot\rho\vec{v}\,v_1 \\ \nabla\ \rho\vec{v}\,v_2 \\ \nabla\ \rho\vec{v}\,v_3 \end{pmatrix} = \nabla(-PI + \bar{\bar{\tau}})$$

③ 能量守恒

内能 ρe

动能 $\frac{1}{2}\rho \vec{v}\cdot\vec{v}$

$$E = \rho e + \frac{1}{2}\rho \vec{v}\cdot\vec{v}$$



小方块

质量交互

$$\Delta\int E\, d\Omega = -\int E(\vec{v}\cdot\vec{n})\,\Delta t\, d\partial\Omega$$

$$+\int \vec{f}\cdot\vec{v} - \vec{q}\cdot\vec{n}\, d\partial\Omega$$

做功 热传导

$$f = \sigma\cdot\vec{n}$$

傅立叶热传导定律： $\vec{q} = -k\nabla T$

$$\frac{\partial E}{\partial t} + \nabla\left((E+p)\vec{v}\right) = \nabla\left(\tau\cdot\vec{v} + k\nabla T\right)$$

推广：简化方程

热方程：$e = c_v T$

$$\frac{\partial T}{\partial t} = \nabla(k\nabla T)$$

对流扩散方程

$$\frac{\partial q}{\partial t} + \nabla(q\vec{v}) = \nabla(k\nabla q)$$

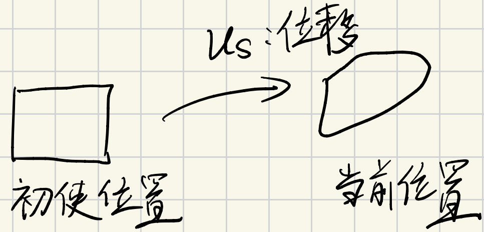
Burgers 方程

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{1}{2}u^2\right) = 0$$

固体方程

动量守恒方程

物质单元



表面应力　　外力

$$\Delta\int \rho_s \dot{u}_s \, d\Omega = \int \sigma\cdot\vec{n} \, d\partial\Omega \, \Delta t + \int f \, d\Omega \, \Delta t$$

$$\frac{d}{dt}\int \rho_s \dot{u}_s \, d\Omega = \int \nabla\sigma \, d\Omega + \int f \, d\Omega$$

$$\frac{d}{dt}\int \rho_s \, d\Omega = 0$$ （质量守恒）

$$\rho_s \ddot{u}_s = \nabla\sigma + f$$ （当前位置）

应力张量 σ

应变张量 $\epsilon = \frac{1}{2}(\nabla u_s + \nabla u_s^T)$

本构关系

$$\sigma = \sigma(\varepsilon)$$

推广：简化方程

波动方程

$$\ddot{u} = \Delta u$$