HOMEWORK 6 - DUE APRIL 16TH, IN CLASS.

Exercise 0.1. *Read the proof of the Omitting types theorem in Marker's* Model theory: An Introduction, *i.e. Theorem 4.2.3.*

Exercise 0.2. *Let $\mathcal{L}$ be a countable language and κ an infinite cardinal. Let M be an $\mathcal{L}$-structure such that $|M| > \kappa$. Prove that the following are equivalent:*

1. *For every $A \subseteq M$ such that $|A| < \kappa$ and $p \in S_1(A)$, there exists some $a \in M$ such that $a \models p$.*
2. *For every $A \subseteq M$ such that $|A| < \kappa$ and $n \geq 1$ where $p \in S_n(A)$, there exists some $\bar{a} \in M^n$ such that $\bar{a} \models p$.*

Exercise 0.3. *Let $\mathcal{L}$ be a countable language and M an infinite $\mathcal{L}$-structure and $A \subseteq M$ such that $|A| \leq \aleph_0$. Prove that for any finite tuple $\bar{x} = (x_1, ..., x_n)$, $|A| \leq |S_{\bar{x}}(A)| \leq 2^{\aleph_0}$.*

Exercise 0.4. *Does there exists a countable $\mathcal{L}$-structure M such that $|S_1(\emptyset)| = 1$ while $|S_2(\emptyset)| \geq \aleph_0$? If yes, provide example and justify. If no, prove.*

Exercise 0.5. *Let $\mathcal{L}$ be a countable language and M an infinite $\mathcal{L}$-structure. Suppose that for every $M \prec N$ and $a, b \in N \backslash M$, there exists an automorphism $\sigma : N \to N$ such that $\sigma(a) = b$ and which fixes M pointwise (i.e. for every $c \in M$, $\sigma(c) = c$). Determine $|S_1(M)|$ and prove your claim.*

Exercise 0.6. *Here we count types. You need some type of justification for each answer.*

1. *Consider $M = (\mathbb{Q}, <)$. Find the size of $S_1(\mathbb{Q})$ and $S_1(\emptyset)$. Justify your claim.*
2. *Consider $M = (\mathbb{N}, <, S)$. Find the size of $S_1(\mathbb{N})$. Justify your claim.*
3. *Consider $M = (\mathbb{Z}; 0, +)$. Find the size of $S_1(\emptyset)$. Justify your claim.*
4. *Consider $M = (\mathbb{G}; R)$ be a model of the random graph. Find the size of $S_1(A)$ where A is any countable set. Justify your claim.*

Exercise 0.7. *If you have time, read* The recent history of model theory *by Enrique Casanovas. See* `http://www.ub.edu/modeltheory/documentos/HistoryMT.pdf`