HOMEWORK 7 - DUE MAY 7TH, IN CLASS.

Exercise 0.1. *Let T be a complete countable theory. Suppose that $\mathcal{U}$ and $\mathcal{V}$ are two saturated models of T such that $|\mathcal{U}| = |\mathcal{V}|$. Prove that $\mathcal{U} \cong \mathcal{V}$.*

Exercise 0.2. *We prove some properties about the category of models of a complete first order theory. Again, let T be a complete countable theory.*

1. *(Joint embedding) Prove that for any $M, N \models T$ there exists some $K \models T$ and elementary embeddings $f : M \to K$ and $g : N \to K$.*
2. *(Amalgamation Property) Suppose that $M_0, N_1, N_2 \models T$ and $f_1 : M_0 \to N_1$ and $f_2 : M_0 \to N_2$ are elementary embeddings. Prove that there exists a model K and elementary embeddings $g_1 : N_1 \to K$ and $g_2 : N_2 \to K$ such that $g_1 \circ f_1 = g_2 \circ f_2$.*

Exercise 0.3. *Let T be the theory of the random graph. Determine $RM(T)$.*

Exercise 0.4. *Let $X \subseteq \mathcal{U}^n$ and $Y \subseteq \mathcal{U}^m$ be definable subsets of our monster model. We say a map $f : X \to Y$ is a definable bijection if f is a bijection and the set $\{(x, y) : y = f(x)\} \subseteq \mathcal{U}^{n+m}$ is definable. Prove that if there exists a definable bijection from X to Y, then $RM(X) = RM(Y)$.*

Exercise 0.5. *Let $\theta(\bar{x}, \bar{y})$ be an $\mathcal{L}$-formula without parameters. Suppose that $\bar{a}, \bar{b} \in \mathcal{U}^{|\bar{y}|}$ and $\mathrm{tp}(\bar{a}/\emptyset) = \mathrm{tp}(\bar{b}/\emptyset)$. Prove that $RM(\theta(\bar{x}, \bar{a})) = RM(\theta(\bar{x}, \bar{b}))$. Hint: Use the fact your are working in a monster model.*

Exercise 0.6. *Give an example of a theory T such that $RM(T) = \omega$. Keep it simple. Try working in a language with only unary predicates. Argue your theory works.*

Exercise 0.7. *Recall that we defined Morley rank relative to a monster model. This exercise shows what goes wrong when we consider an arbitrary model of a first order theory. Consider the structure $M = (\mathbb{N}, <)$ and let T be the corresponding theory.*

1. *Compute the Morley rank of $x = x$ relative to this structure, i.e. pretend that M is our monster model and compute the Morley rank here.*
2. *Compute the actual Morley rank of the theory.*