HOMEWORK 9 - DUE BEFORE JUNE 4TH IN CLASS

Exercise 0.1. Let $D = \{(a,b) \in \mathbb{R}^2 : a^2 + b^2 \leq 1\}$ and $B = \{(a,b) \in \mathbb{R}^2 : a^2 + b^2 = 1\}$. We define a relation $R(x,y) \subseteq D \times \mathcal{P}_{fin}(B)$ where $\models R(b,A)$ if and only if b is in the convex hull of A, i.e. $b = \sum_{a \in A} t_a a$ where for each $a \in A$, $t_a \in [0,1]$ and $\sum_{a \in A} t_a = 1$. Determine if this relation is stable.

Exercise 0.2. Suppose that $(a_i)_{i \in I}$ and $(b_i)_{i \in I}$ are indiscernible over A. Does this imply that $(a_i, b_i)_{i \in I}$ is indiscernible over A?

Exercise 0.3. Fix a field K and consider the language of K-vector spaces, $\mathcal{L} = \{(r_\alpha)_{\alpha \in K}, +, 0\}$ where r_α is a unary function symbol for each $\alpha \in K$. Now, let $K = \mathbb{Q}$ and consider $V = \mathbb{R}$ as a $\mathbb{Q}$-vector space in this language.

(1) Give an explicit indiscernible sequence from V.
(2) Prove your choice of sequence works.

Exercise 0.4. Suppose that T is t.t. Let $(X_i)_{i \in I}$ be an infinite collection of definable sets of $\mathcal{U}^x$ such that for each i, we have $RM(X_i) \geq \alpha$ and additionally, the rank of the intersection of any n sets is strictly less than α. Then there exists an infinite subcollection $(Y)_{j \in J}$ and an integer $2 \leq k \leq n-1$ such that the rank of the intersection of any k-many sets has rank α but the rank of the intersection of any $k+1$ sets has rank strictly less than α. Hint: Ramsey.

Exercise 0.5. Consider the theory T_2 which is the theory of two infinite equivalence classes in the language $\mathcal{L} = E$. We usually consider the theory of infinitely many equivalence relations all with infinitely many class, but here we really just mean 2. Now, there is a unique type $p_0 \in S_x(\emptyset)$. Let $M \subseteq \mathcal{U}$ be a model of T. Determine the Morley degree of p_0 and the number of non-forking extension of q_0 in $S_x(M)$ and be careful.

Exercise 0.6. Suppose that T is t.t. Let $M \subseteq \mathcal{U}$ and suppose that $p \in S_x(M)$ where M is a model of T. Prove that $Md(p) = 1$.

Exercise 0.7. Suppose that T is t.t. Show that

$$|\{p \in S_x(\mathcal{U}) : MR(p) = MR(T)\}| < \aleph_0.$$

Exercise 0.8. Go to `https://www.forkinganddividing.com` and find a section/example from the map that you find interesting. Write a little summary about it.