Fall 2025: Topics in Number Theory: Introduction to p-adic Hodge theory

Instructor: Liang Xiao (肖梁)
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Meeting time: Tuesday 5–6 (odd weeks) and Thursday 7–8
Lecture room: Lecture Building #2, Room 416 (二教 416)
Office hours: by appointment
Webpage: `http://bicmr.pku.edu.cn/~lxiao/25fall/25fall.htm`

Goal of this course: This course gives an introduction to p-adic Hodge theory, including the following topics.

(1) Galois theory for completed valued fields.
(2) Lubin–Tate group and explicit construction of local class field theory.
(3) Scholze's tilting equivalence for perfectoid fields.
(4) Tate's normalized trace and Sen theory for Galois representations.
(5) Fontaine's (φ, Γ)-modules.
(6) Period rings in p-adic Hodge theory: $\mathbb{B}_{\mathrm{dR}}$, $\mathbb{B}_{\mathrm{crys}}$, and etc.
(7) Overconvergent (φ, Γ)-modules and beyond.

We will follow relatively closely Berger's IHP notes in 2010: "Galois representation and (φ, Γ)-modules", available at `https://perso.ens-lyon.fr/laurent.berger/autrestextes/CoursIHP2010.pdf`

Prerequisite:

- Very well versed with algebraic number theory, especially with local fields;
- Familiar with Galois cohomology, especially with local fields.
- Some background on algebraic geometry (Hartshorne Chapter 2 and 3) and some exposure to étale cohomology might be helpful but not necessary.

Grade Distribution:
Homeworks: 60%, due on Thursdays of Week 3, 6, 8, 10, 12, 14, 16, in total 7 times, with lowest grade dropped.
Take-home final exam: 40%, to be announced (probably one or two French-style long problems).

Homework: *Homework problems will be posted on the course webpage.* You are welcome and encouraged to work with other students on the problems, but you should write up your homework independently.

Syllabus (Tentative)

Lecture	Dates	Content
1	9/9	Introduction and motivation
2	9/11	Grothendieck's monodromy theorem, Toolbox of Newton polygon
3	9/18	p-adic functions on annuli, Ax–Sen–Tate Theorem
4	9/23	Lubin–Tate formal groups
5	9/25	Lubin–Tate's construction of local class field theory
Happy National's Day!		
6	10/9	Perfectoid fields and tilting process.
7	10/16	Tilting equivalence.
8	10/21	Tate's normalized trace.
9	10/23	Tate–Sen theory for $\mathbb{C}_p$-representations.
10	10/30	Colmez–Tate–Sen theory, imperfect period rings, and (φ, Γ)-modules.
11	11/4	Galois cohomology in terms of (φ, Γ)-modules.
12	11/6	ψ-operators and Tate duality in terms of (φ, Γ)-modules.
13	11/13	Crystalline and de Rham period rings. (LX away, taught by Yiwen Ding)
14	11/18	p-adic Hodge theory in terms of period rings.
15	11/20	$\mathbb{D}_{\mathrm{dif}}(V)$.
16	11/27	Overconvergent period rings, Colmez–Tate–Sen theory for overconvergent elements. (LX away, taught by Yiwen Ding)
17	12/2	Overconvergent (φ, Γ)-modules.
18	12/4	(φ, Γ)-modules over Robba rings
19	12/11	Berger's functor I.
20	12/16	Berger's functor II.
21	12/18	Cohomology of (φ, Γ)-modules over Robba rings.
22	12/25	Triangulline (φ, Γ)-modules and introduction to global triangulation.
	TBA	**Final Exam**