Errata for "Cohomology of Arithmetic Family of (φ, Γ)-modues

- **Remark 2.2.16** (pointed out to us by Rebecca Bellovin) The statement that "[Conjecture 2.2.15] is also known for the essential image of the functor $\mathbf{D}_{\mathrm{rig}}$" is not quite accurate. That would become true if we weakened Conjecture 2.2.15 by only requiring the conclusion after replacing K with an unspecified finite extension (because this is needed to ensure that a free representation turns into a free (φ, Γ)-module; see the technical definition of L just before Proposition 4.2.8 of the paper "Familles de représentations de de Rham et monodromie p-adique" of L. Berger and P. Colmez).
- **Theorem 6.3.9** paragraph 2 of the proof, line 2-3, remove "(resp. $[-1,2]$)", i.e. $\mathrm{C}^{\bullet}_{\varphi,\gamma_K}(M^\vee(\delta)/t_\sigma)$ is quasi-isomorphic to some complex of locally free coherent sheaves concentrated in degree $[0,2]$ (as opposed to just $[-1,2]$); this is exactly the statement of Corollary 6.3.3.
- **Remark 6.3.14** This is just to clarify a subtlety at the last line of page 1109. Write $\kappa_z[\![\varpi_z]\!]$ for the ring of completion of X at z. For simplicity we assume $K = \mathbb{Q}_p$. Suppose that the localization of Q at z is $\mathcal{R}_{\kappa_z[\![\varpi_z]\!]}(\delta_2)/(\varpi_z^{i_1}, \varpi_z^{i_2}t^{j_1}, \dots, \varpi_z^{i_m}t^{j_{m-1}}, t^{j_m})$, with $i_1 > \cdots > i_m$ and $j_1 < \cdots < j_m$. Then $Q_z = \mathcal{R}_{\kappa_z}(\delta_{2,z})/(t^{j_m})$ and

$$Q[\varpi_z] := \left\{m \in Q; \varpi_z m = 0\right\} \cong \frac{\mathcal{R}_{\kappa_z}(\delta_{2,j})}{(t^{j_1})} \oplus \frac{\mathcal{R}_{\kappa_z}(\delta_{2,j}x^{j_1})}{(t^{j_2-j_1})} \oplus \cdots \oplus \frac{\mathcal{R}_{\kappa_z}(\delta_{2,j}x^{j_{m-1}})}{(t^{j_m-j_{m-1}})}.$$

The module $\mathrm{Tor}_1^X(Q, \kappa_z)$ is (by definition of Tor) $Q[\varpi_z]$. But it is, as argued in the paper, at the same time equal to a quotient of $\mathrm{Ker}\mu_z$. So it is monogenic. This forces the number m above to be 1, and the localization of Q at z simply takes the form of $\mathcal{R}_{\kappa_z[\![\varpi_z]\!]}(\delta_2)/(\varpi_z^i, t^k)$. This argument generalizes to the case with general K and shows that the localization of Q at z is a direct sum of $\mathcal{R}_{\kappa_z[\![\varpi_z]\!]}(\pi_K)(\delta_2)/(\varpi_z^{i_\sigma}, t^{k_\sigma})$ over all σ's and $i_\sigma \in \mathbb{N}$ whenever $k_{z,\sigma} \neq 0$. In particular, all $k_{z,\sigma,n}$ appearing on page 1110 are independent of n.