

Reading seminars on Galois representations and (φ, Γ) -modules

While ℓ -adic representations of the Galois group of a finite extension K of \mathbb{Q}_p with $\ell \neq p$ is “easier” to understand thanks to Grothendieck’s monodromy theorem, the p -adic representation of Gal_K is much more complicated yet interesting. In this seminar series, we will follow a course taught by Laurent Berger at IHP in 2010,

- Galois representations and (φ, Γ) -modules, available at <http://perso.ens-lyon.fr/laurent.berger/other.php>

This course covers the basics of p -adic Hodge theory and the theory of (φ, Γ) -modules, which is extremely important in the student of p -adic representations of Gal_K .

Other helpful references are listed at the end of this file.

The seminar runs twice a week, 2 hours each, covering two lectures.

Talk schedule (Tentative)

Dates & time	Speaker	Content
7/14, 20, 3-5pm	Xiao Liang	Lectures 2–5.
7/21, 3-5pm	Jiang Jiedong	Lectures 6–8. Carefully explain the Witt vector construction. No need to spend too much time defining Galois cohomology, but explain the proof of Proposition 7.4 carefully. Discuss Lecture 8 carefully.
7/27, 3-5pm	Qiu Tian	Lectures 9 and 10. Make all the computation explicitly and carefully.
7/28, 3-5pm	Peng Hao	Lectures 11–13. We have already learned Lubin–Tate groups in Yiwen’s course. Summarize that part; but spend some time on the periods of Lubin–Tate groups. Lecture 13 is very important
8/3, 3-5pm	Julian Bayerl	Lectures 14 and 15. Starting from here, we are entering the world of p -adic Hodge theory. State the tilting equivalence and almost purity for perfectoid algebras ([Scholze, Theorems 5.2, 5.25]).
8/4, 3-5pm	Guo Jingbang	Lectures 16 –18. The notion of (φ, Γ) -modules appears for the first time.
8/10, 3-5pm	Liao Yuxuan	[Colmez, §5.1–5.3.1]. We switch to Colmez’ notes to cover Galois cohomology computation
8/11, 3-5pm	Aditya Karnataki	[Colmez, rest of §5, §6]. Finish the proof for Euler–Poincaré characteristic; Iwasawa cohomology (skip the part about interpretation by measure).
8/17, 3-5pm	Chen Zekun	Lectures 19 and 20. This is the first hard-core theorem: Sen’s theory
8/24, 25, 3-5pm	Wang Yupeng	Lectures 21–25. (2 lectures) Overconvergent theory

8/31, 3-5pm	Luo Jinyue	Lectures 26–28. [Berger2, §5.3]. Introduce de Rham representations. Recover D_{Sen} and D_{dR} using (φ, Γ) -modules.
9/1, 7, 3-5pm	Léo Poyeton	Lectures 29–31. [Berger2, §3.1-3.2]. Introduce crystalline and semi-stable representations. Recover D_{cris} and D_{st} using (φ, Γ) -modules.
9/8, 3-5pm	Ji Yibo	Slope filtrations. [Kedlaya, §1-2.2] Introduce the notion and basic properties of φ -modules over the Robba ring. State the classification of φ -modules over the extended Robba ring and apply it to deduce the slope filtration theorem.
9/14, 3-5pm	Yu Jiahong	Theorem A. [Berger3]. Sketch the proof of the so called “Theorem A” (weakly admissible \implies admissible).
9/15, 3-5pm	Liu Ruochuan	Theorem B. [Berger2, §5]. Sketch the proof of the so called “Theorem B” (de Rham \implies potentially semi-stable).

REFERENCES

- [Berger] L. Berger, An introduction to the theory of p -adic representations, Geometric aspects of Dwork theory. Vol. I, 255–292, Walter de Gruyter, 2004.
- [Berger2] L. Berger, Représentations p -adiques et équations différentielles, Inv. Math. 148, 2002, 219-284.
- [Berger3] L. Berger, Équations différentielles p -adiques et (ϕ, N) -modules filtrés, Astérisque No. 319 (2008), 13–38.
- [Brinon-Conrad] O. Brinon and B. Conrad, CMI summer school notes on p -adic Hodge theory.
- [Colmez] P. Colmez, Fontaine’s rings and p -adic L-functions (lectures at Tsinghua), available at <https://webusers.imj-prg.fr/~pierre.colmez/Enseignement.html>
- [Kedlaya] Slope filtrations for relative Frobenius, Astérisque 319 (2008), 259-301.
- [Scholze] P. Scholze, Perfectoid spaces, Publ. math. de l’IHÉS 116 (2012), no. 1, 245-313.