

POSSIBLE PROJECTS FOR HODGE THEORY AND COMPLEX ALGEBRAIC GEOMETRY

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If you make your own choice, make sure to discuss it with the instructor first. A general rule for the choice, you should be able to explain the material in some depth (that is, talk about relevant definitions/concepts and discuss some theorems with sketch of proof or some key elements) to your classmates in 50 minutes, assuming only what we have talked about in the semester.

Some of the topics below contain more than what could be a class project. Talk to the instructor first about what should be the appropriate work load.

You need to write a report (prepared using \LaTeX) and you may need to present your project in class for the evaluation.

- (1) GAGA principle, comparison between algebraic varieties and their analytification. Géométrie algébrique et Géométrie analytique, by Serre, or by Raynaud, or others.
- (2) Kodaira problem, [Voi04], or the Bourbaki report by Huybrechts (Séminaire BOURBAKI Novembre 2005 58ème année, 2005-2006, no 954).
- (3) Mixed Hodge structure [Voi02, II.8], [Voi03], Deligne Théorie de Hodge.
- (4) Algebraic cycles, Abel-Jacobi map. Hodge conjecture. [Voi02, IV]
- (5) Picard-Lefschetz theory, monodromy, Noether-Lefschetz theorem. [Voi03, I.1-3]
- (6) Hodge filtration of hypersurfaces and applications [Voi03, II.6].
- (7) Vanishing theorem: proof using Hodge theory (Kollár, Part III, Shafarevich maps and automorphic forms, Princeton University Press) or singular Hermitian metric with L^2 -estimate (Demailly, singular Hermitian metrics on positive line bundles, Springer lecture notes vol. 1507), generalizations, applications.
- (8) Positivity from variation of Hodge structure, Fujita's theorem (Fujita, On Kähler fiber space over curves, J. Math. Soc. Japan, Vol. 30, No. 4.), applications.
- (9) Mirror symmetry and variation of Hodge structure, Picard-Fuchs equation, quintic threefold, mirror theorem. Mirror symmetry, Clay Math and AMS, Chap. 6.
- (10) Grassmann and Schubert calculus. Griffiths Harris 1.5
- (11) Abelian variety and jacobian of a curve, Griffiths Harris 2.6, 2.7

REFERENCES

- [Voi02] Claire Voisin. *Hodge theory and complex algebraic geometry. I*, volume 76 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2002. Translated from the French original by Leila Schneps.

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- [Voi03] Claire Voisin. *Hodge theory and complex algebraic geometry. II*, volume 77 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2003. Translated from the French by Leila Schneps.
- [Voi04] Claire Voisin. On the homotopy types of compact Kähler and complex projective manifolds. *Invent. Math.*, 157(2):329–343, 2004.

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