RESEARCH HIGHLIGHTS (2016-2020)

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1. Arithmetic of rationally connected varieties. A major theme of my research is to develop geometric techniques to understand arithmetic questions of rationally connected varieties, especially about rational points and zero cycles.

1.1 Rational points. The celebrated Hasse-Minkowski theorem says that there is a rational point of a quadric defined over a global field, if and only if there is a rational point over the completion at each places of the global field. Many works have been done to extend this theorem (a.k.a. Hasse principle) to other cases. A general conjecture due to Colliot-Thélène predicts that the Brauer-Manin obstruction is the only obstruction to Hasse principle for (separably) rationally connected varieties defined over global fields. Thus, one expects that Hasse principle holds for smooth Fano complete intersections of dimension at least 3.

Analytic and algebraic methods have been developed to tackle this problem. In [Tia17], I make some progress on this problem, using a completely different geometric approach. This result is so far the strongest result on Hasse principle for the rationally connected varieties over global function fields

Once we understand Hasse principle, the next problem is weak approximation. In a joint work with Letao Zhang [TZ18], we study weak approximation problems for certain varieties defined over global function fields using geometric methods.

1.2 Zero cycles. Very recently, I studied zero cycles on rationally connected varieties defined over different fields.

For rationally connected varieties defined over an non-algebraically closed field, one would like to understand the Chow group of zero cycles via the map to étale cohomology. When the base field is $\mathbb{C}(\!(t)\!)$, for some scattered examples, one can show that the Chow group of zero cycles is isomorphic to $\mathbb{Z}$ via the degree map. But the general situation remains mysterious and experts seem to think that there is no general results like the special cases. Much to the surprise of experts, in a very recent preprint [Tia20b], I introduce techniques from the minimal model program to study this question, and provide very strong evidence that the Chow group of zero cycles is **always** isomorphic to $\mathbb{Z}$.

One could also ask whether or not local-global principle is true for zero cycles, and whether or not Brauer-Manin obstruction is the only obstruction for local-global principle to hold. My work on such problems [Tia20a] proves that the Brauer-Manin obstruction is the only obstruction for rational surfaces, and some other class of rationally connected varieties defined over global function fields.

2. Motivic crepant resolution conjecture, McKay correspondence. Hyperkähler manifolds are higher dimensional analogues of $K3$ surfaces. They are also building blocks of varieties with trivial first Chern classes. In joint works with Lie Fu [FT17] and Lie Fu-Charles Vial [FTV19], we study the Chow motive of such manifolds. Part of our motivation comes from string theory, namely, the crepant resolution conjecture. The goal of our project is to understand how much of this conjecture is true on the level of Chow motives, and to use this knowledge to study the Chow ring of hyperkähler manifolds. There are several applications of this type of results, e.g., multiplicative decomposition of direct images, splitting of the Chow motive of hyperkähler manifolds, Beauville-Voisin conjectures, etc. In joint work with Lie Fu, we also study multiplicative motivic McKay correspondences [FT19].

References

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