Homework for "Algorithms for Big-Data Analysis"

Beijing International Center for Mathematical Research
Peking University

February 17, 2024

Note: Please write up your solutions independently. If you get significant help from others, write down the source of references. A formal mathematical proof for all your claims is required.

1. Consider the integer programming problem:

$$\begin{aligned} \max \quad & x_1 + 2x_2 \\ \text{s.t.} \quad & -3x_1 + 4x_2 \leq 4 \\ & 3x_1 + 2x_2 \leq 11 \\ & 2x_1 - x_2 \leq 5 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \text{ integer.} \end{aligned}$$

   (a) What is the optimal cost of the linear programming relaxation? What is the optimal cost of the integer programming problem?

   (b) What is the convex hull of the set of all solutions to the integer programming problem?

   (c) Illustrate how the Gomory cutting plane algorithm would work. Give the first cut.

   (d) Solve the problem by branch and bound. Solve the linear programming relaxations graphically.

   (e) Suppose you dualize the constraint $-3x_1 + 4x_2 \leq 4$. What is the optimal value Z_D of the Lagrangian dual?

   (f) Suppose you dualize the constraint $2x_1 - x_2 \leq 5$. What is the optimal value Z_D of the Lagrangian dual?