

Project on “Optimal Transport”

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Consider the optimal transport problem:

$$(1) \quad \begin{aligned} \min_{\pi \in \mathbb{R}^{m \times n}} \quad & \sum_{i=1}^m \sum_{j=1}^n c_{ij} \pi_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^n \pi_{ij} = \alpha_i, \quad \forall i = 1, \dots, m, \\ & \sum_{i=1}^m \pi_{ij} = \beta_j, \quad \forall j = 1, \dots, n, \\ & \pi_{ij} \geq 0, \end{aligned}$$

where c , α and β are given, and $\sum_{i=1}^m \alpha_i = \sum_{j=1}^n \beta_j = 1$, $\alpha \geq 0$ and $\beta \geq 0$.

1. Solve (1) by calling mosek and gurobi **directly** in Matlab or python. The package “CVX” is **not allowed** to use here. Compare the performance between the simplex methods, the interior point methods and the network simplex method (if it is available).
2. Write down and implement the Sinkhorn method.
3. (Optional) Write down and implement a first-order method, for example, the alternating direction method of multipliers.
4. Test problems:
 - Generate the data c , α and β using the following code and images:
http://faculty.bicmr.pku.edu.cn/~wenzw/bigdata/gen_ot_data.m
<http://faculty.bicmr.pku.edu.cn/~wenzw/bigdata/source.png>
<http://faculty.bicmr.pku.edu.cn/~wenzw/bigdata/dest.png>
 - Choose two other images based on your own preference and generate the data.
5. Requirement:
 - (a) Compare the efficiency (cpu time) and accuracy (checking optimality condition) of different methods.
 - (b) Prepare a report including
 - detailed answers to each question

- numerical results and their interpretation
- (c) Pack all of your codes in one file named as “projot-name-ID.zip” and send it to TA:
pkupt@163.com
- (d) If you get significant help from others on one routine, write down the source of references at the beginning of this routine.