# Project for “Convex Optimization”

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## 1 Projection to Doubly Stochastic Matrices

A matrix $X \in \mathbb{R}^{n\times n}$ is a called a doubly stochastic matrices if

$$\begin{aligned}\sum_{i=1}^n X_{ij} &= 1, \quad j = 1,\ldots,n,\\ \sum_{j=1}^n X_{ij} &= 1, \quad i = 1,\ldots,n,\\ X &\geq 0.\end{aligned}$$

Denote the set of the doubly stochastic matrices by

$$\mathcal{D} = \{X \in \mathbb{R}^{n\times n} \mid Xe = X^\top e = e, X \geq 0\},$$

where $e$ is a vector of all ones.

Given $Z \in \mathbb{R}^{n\times n}$, the projection of $Z$ to $\mathcal{D}$ is

$$\min \quad \frac{1}{2}\|X - Z\|_F^2, \qquad \text{s.t. } X \in \mathcal{D}. \tag{1.1}$$

1. Solve (1.1) using CVX by calling different solvers: sdpt3, mosek, gurobi. Test matrices:
   - n = 50; A = rand(n);
   - n = 50; A = randn(n);
   - Z1 and Z2 in `http://bicmr.pku.edu.cn/~wenzw/courses/test_double_stochastic_matrix.mat`
2. Solve (1.1) by calling the quadratic programming and SOCP solvers (if they are available) in sdpt3, mosek, gurobi directly. Test matrices are the same as question 1.
3. Check the accuracy of solutions on Z1 and Z2 using different tolerance levels: $10^{-6}$, $10^{-9}$ and $10^{-12}$. Measure the differences of the solutions using “norm”.
4. Derive the dual problem of (1.1).

5. Write down a gradient or proximal gradient algorithm for solving (1.1). Implement this algorithm in Matlab and test matrices in question 1.

6. Write down a Nesterov acceleration method for the gradient or proximal gradient method in question 5. Implement this algorithm in Matlab and test matrices in question 1.

7. Write down two version of the alternating direction methods of multipliers (ADMM) for solving (1.1). Implement them in Matlab and test matrices in question 1.

8. Compare efficiency (cpu time) and accuracy (checking optimality condition) of the above algorithms on the random examples by varying $n = 100, 200, \ldots, 500$.

9. The quadratic assignment problem (QAP) is

$$\min_{X \in \mathbb{R}^{n \times n}} \quad \mathrm{tr}(A^\top X B X^\top), \text{ s.t. } X^\top X = I, X \geq 0. \tag{1.2}$$

A relaxation to QAP is

$$\min_{X \in \mathbb{R}^{n \times n}} \quad \mathrm{tr}(A^\top X B X^\top), \text{ s.t. } X \in \mathcal{D}. \tag{1.3}$$

- Design a first-order method to solve (1.3).
- Test matrices: Chr12a, Bur26a, Esc16a, Had18 in QAPLIB: `http://anjos.mgi.polymtl.ca/qaplib/data.d/qapdata.tar.gz`
- Compare the solution of (1.3) with the optimal solution of (1.2) which are available at `http://anjos.mgi.polymtl.ca/qaplib/inst.html`