Homework 5 for "Convex Optimization"

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1 Algorithms for ℓ_1 minimization

Consider the ℓ_1-regularized problem

$$\min_x \quad \frac{1}{2}\|Ax-b\|_2^2 + \mu\|x\|_1, \tag{1.1}$$

where $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$ and $\mu > 0$ are given. Test matrices:

```
seed = 97006855;
ss = RandStream('mt19937ar','Seed',seed);
RandStream.setGlobalStream(ss);
n = 1024;
m = 512;
A = randn(m,n);
u = sprandn(n,1,0.1);
b = A*u;
mu = 1e-3;
```

See `http://bicmr.pku.edu.cn/~wenzw/courses/Test_l1_regularized_problems.m`

1. Solve (1.1) using CVX by calling different solvers mosek and gurobi.
2. First write down an equivalent model of (1.1) which can be solved by calling mosek and gurobi directly, then implement the codes.
3. First write down, then implement the following algorithms in Matlab (or Python):
   (a) Projection gradient method by reformulating the primal problem as a quadratic program with box constraints.
   The projection gradient method is a special version of the proximal gradient method. Read
   `http://bicmr.pku.edu.cn/~wenzw/opt2015/lect-proxg.pdf`
   (b) Subgradient method for the primal problem.
   Read the subgradient method in
   `http://bicmr.pku.edu.cn/~wenzw/opt2015/lect-sgm.pdf`

(c) Gradient method for the smoothed primal problem.
Read the smoothing technique in
`http://bicmr.pku.edu.cn/~wenzw/opt2015/Smoothing.pdf`

(d) Fast (Nesterov/accelerated) gradient method for the smoothed primal problem.
Read the acceleration techniques in
`http://bicmr.pku.edu.cn/~wenzw/opt2015/slides-fgrad.pdf`

(e) Proximal gradient method for the primal problem.
Read `http://bicmr.pku.edu.cn/~wenzw/opt2015/lect-proxg.pdf`

(f) Fast proximal gradient method for the primal problem.
Read the acceleration techniques in
`http://bicmr.pku.edu.cn/~wenzw/opt2015/slides-fgrad.pdf`

(g) Augmented Lagrangian method for the dual problem.
Read the augmented Lagrangian method in
`http://bicmr.pku.edu.cn/~wenzw/opt2015/lect-prox-point.pdf`

(h) Alternating direction method of multipliers for the dual problem.
Read the augmented Lagrangian method in
`http://bicmr.pku.edu.cn/~wenzw/opt2015/lect-prox-point.pdf`

(i) Alternating direction method of multipliers with linearization for the primal problem.
Read the ADMM in `http://bicmr.pku.edu.cn/~wenzw/opt2015/lect-admm.pdf`.
Read the ADMM with a single gradient (or proximal graident) step in pages 15 and 16 in
`http://bicmr.pku.edu.cn/~wenzw/opt2015/lect-admm-part2.pdf`

(j) Proximal point method for the dual problem

(k) Block coordinate method for the primal problem

(l) Semi-smooth Newton method

`http://www.deeplearningbook.org/contents/optimization.html`

4. Requirement:

(a) The interface of each method should be written in the following format

```
[x, iter, out] = method_name(x0, A, b, mu, opts);
```

Here, x0 is a given input initial solution, A, b and mu are given data, opts is a struct which stores the options of the algorithm, iter is the number of iterations when the termination condition of the algorithm is satisfied, out is a struct which saves all other output information.

(b) Compare the efficiency (cpu time) and accuracy (checking optimality condition) in the format as
`http://bicmr.pku.edu.cn/~wenzw/courses/Test_l1_regularized_problems.m`

(c) Prepare a report including

- detailed answers to each question
- numerical results and their iterpretation

(d) Pack the report and all of your codes in one file named as “l1-StudentID-date.zip” and send it to TA: pkuopt@163.com

(e) If you get significant help from others on one routine, write down the source of references at the beginning of this routine.

(f) Due date

- Nov. 17: 1, 2, 3 (a)
- Dec. 1: 3 (b), (c)
- Dec. 15: 3 (d), (e), (f)
- Jan. 5: 3 (g), (h), (i)
- optional: (j), (k), (l)