EXERCISE SHEET #1

Let $\mathbb{B}^2$ denote the the unit open disk of $\mathbb{R}^2$.

Execise 0.1. *Prove that the subgroup of $M^+(\hat{\mathbb{R}}^2)$ leaving $\mathbb{B}^2$ invariant is isomorphic to the following subgroup of $PSL(2,\mathbb{C})$:*

$$\left\{ \frac{az+\bar{c}}{cz+\bar{a}} : a, c \in \mathbb{C}, |a|^2 - |c|^2 = 1 \right\}$$

Execise 0.2. *Let $\phi = \frac{az+b}{cz+d} \in PSL(2,\mathbb{C})$ where $a, b, c, d \in \mathbb{C}, ad - bc = 1$. Prove*

$$\frac{|\phi(z) - \phi(w)|}{|z - w|} = \sqrt{|\phi'(z)|}\sqrt{|\phi'(w)|}.$$

Let $\mathbb{H}^n = \{x \in \mathbb{R}^n : x_n > 0\}$ be the upper half space equipped with hyperbolic metric ρ (induced from Riemannian metric $\frac{|dx|}{x_n}$).

Execise 0.3 (Nearest projection)**.**

(1) *Let x be in $\mathbb{H}^n$. Show that the nearest point to x on the positive n-th axis is $|x|e_n$ and we have*

$$\cosh \rho(x, |x|e_n) = \frac{|x|}{x_n}$$

(2) *Let π be the nearest point projection of a point $x \in \mathbb{H}^n$ onto the positive n-th axis defined by $\pi(x) = |x|e_n$. Prove that for all $x, y \in \mathbb{H}_n$, we have*

$$\rho(\pi(x), \pi(y)) \leq \rho(x, y)$$

with equality if and only if either $x = y$ or x and y lie on the n-th axis.

Execise 0.4. *Suppose that $\phi \in M(\hat{\mathbb{R}}^{n+1})$ preserves $\mathbb{H}^{n+1}$. Then ϕ is the Poincare extension of some $\psi \in \hat{\mathbb{R}}^n$.*