EXERCISE SHEET #7

Exercise 0.1 (Generating set)**.** *Let G be a subgroup of $I(\mathbb{H}^n)$ with a locally finite fundamental domain R. Prove that G is generated by the set $\{g \in G : g\bar{R} \cap \bar{R} \neq \emptyset\}$.*

Exercise 0.2 (Co-compact Kleinian groups)**.** *Let G be a discrete subgroup of $I(\mathbb{H}^n)$ with a locally finite fundamental domain R. If $\bar{R}$ is compact, then*

(1) *the limit set ΛG is $\partial_\infty \mathbb{H}^n$.*
(2) *There is a uniform lower bound on the injective radius of any point at $\mathbb{H}^n$.*
(3) *G contains no parabolic elements.*

Strictly invariant subsets. A subset A of $\mathbb{H}^n$ is called *strictly invariant* under a discrete group G if $gA \cap A \neq \emptyset$ for $g \in G$ implies $gA = A$. Let $\pi : \mathbb{H}^n \to \mathbb{H}^n/G$ be the natural projection map.

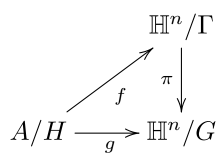
Exercise 0.3. *Let $H := \{g \in G : gA = A\}$ be the stabilizer of A. Consider the space A/H with quotient topology and $\pi(A) \subset \mathbb{H}^n/G$ with subspace topology. Prove that A/H and $\pi(A)$ are homeomorphic if either*

(1) *A is open in $\mathbb{H}^n$, or*
(2) *A/H is compact.*

Separability. Recall that a subgroup H of a group G is called *separable* if H is closed in the profinite topology on G.

Let $G < I(\mathbb{H}^n)$ be a discrete group without torsions. Let $A \subset \mathbb{H}^n$ be a subset invariant under a subgroup H of G (i.e.: $\forall h \in H, hA = A$).

For subgroups $H \subset \Gamma \subset G$, we define $g : Ha \in A/H \mapsto Ga \in \mathbb{H}^n/G$, $f : Ha \in A/H \mapsto \Gamma a \in \mathbb{H}^n/\Gamma$ and $\pi : \Gamma x \in H^n/\Gamma \mapsto Gx \in H^n/G$. Then the following diagram commutes



Exercise 0.4. (1) *Prove that the above maps f, g, π are covering maps.*
(2) *If A/H is compact with quotient topology, show that there exists a finite index subgroup $H \subset \Gamma$ of G such that the map $f : A/H \to \mathbb{H}^n/\Gamma$ is a topological embedding of A/H into $\mathbb{H}^n/\Gamma$.*

Remark. This exercise says that an immersion g of a compact space A/H into $\mathbb{H}^n/G$ can be lifted to an embedding in a finite cover, when H is separable.

A theorem of Scott says that any finitely generated subgroup in a Fuchisan group is separable. As a corollary, we can remove the self-intersection of any closed essential curve on a surface in a finite cover.