EXERCISE SHEET #8

Let $PSL(2,\mathbb{Z}) = \{\frac{az+b}{cz+d} : a,b,c,d \in \mathbb{Z}, ad-bc=1\}$.

Exercise 0.1. *Let $S(z) = \frac{-1}{z}$ and $R(z) = \frac{z-1}{z}$. Prove that every elliptic element in $PSL(2,\mathbb{Z})$ is conjugated to either S or R.*

Exercise 0.2. *Let G be a discrete elementary group in $I(\mathbb{H}^n)$. Prove that $\mathbb{H}^n/G$ has infinite volume.*

Exercise 0.3. *Let R be a locally finite fundamental domain for a discrete group G in $I(\mathbb{H}^n)$. Prove that*

(1) *for any $x \in \partial R$, the set $Gx \cap \partial R$ is finite..*
(2) *If $R = D(a)$ is the Dirichlet domain centered at a point $a \in \mathbb{H}^n$, then any two points in $Gx \cap \partial R$ have the same distance to a.*

Let C be a closed convex subset in $\mathbb{H}^n$. Denote by $\bar{C}^\infty$ the the Euclidean closure of C in $\mathbb{E}^n$ when $\mathbb{H}^n$ is realized as the ball model in $\mathbb{E}^n$. Then $\partial_\infty C := \bar{C}^\infty \setminus C$ is the boundary of C at the infinity.

Exercise 0.4. *Let G be a discrete subgroup of $I(\mathbb{H}^n)$ with a locally finite, convex, fundamental domain R. Let $\bar{R}$ be the closure of R in $\mathbb{H}^n$ and ΩG be the discontinuity domain. Prove that*

(1) *The interior of $\partial_\infty \bar{R}$ in S^{n-1} is contained in ΩG.*
(2) *$\{gR : g \in G\}$ is a locally finite collection of subsets in $\mathbb{H}^n \cup \Omega G$.*
(3) *Let $K = \Omega G \cap \partial_\infty \bar{R}$. Then $\cup\{gK : g \in G\} = \Omega G$.*
(4) *Conclude that if $\Omega G \neq \emptyset$, then $\mathbb{H}^n/G$ has infinite volume.*