EXERCISE SHEET #2

We call an isometric action of a group G on a metric space X is *co-bounded* if there exists a bounded set K such that $G \cdot K = X$.

Exercise 0.1. *Suppose G acts by co-boundedly on a length space (X, d). Fix a basepoint $o \in X$. Then there exists a (possibly infinite) generating set S of G such that the map*

$$(G, d_S) \to (Go, d),\ g \mapsto go,$$

is a G-equivariant quasi-isometric map.

Exercise 0.2. *Let $d \geq 3$ be an integer. Prove that any two trees with vertices of degree between 3 and d are quasi-isometric.*

Exercise 0.3. *Prove that finite presentability is a quasi-isometric invariant: Assume that two finitely generated groups G and Γ are quasi-isometric. If G is finitely presentable, then Γ is finitely presentable.*

We consider the set of all quasi-isometries of X. Two quasi-isometries $\phi, \psi : X \to X$ are called *equivalent* if they differ by a bounded constant: $||\phi - \psi||_\infty < \infty$. Denote by $QI(X)$ the set of equivalent classes of quasi-isometries of X.

Exercise 0.4. *The set $QI(X)$ with the composition operation is a group. Moreover, there exists a homomorphism from the isometry group $Isom(X)$ of X into the group $QI(X)$.*

Exercise 0.5. *Suppose two metric spaces X, Y are quasi-isometric. Then $QI(X)$ is isomorphic to $QI(Y)$ (given by conjugating the isometric actions on X).*