EXERCISE SHEET #5

Suppose that G acts properly on a proper geodesic δ-hyperbolic space.

Execise 0.1. *Let h be a loxodromic element. Then the stabilizer in G of the two fixed points of h is virtually cyclic.*

Execise 0.2. *If h, k are two loxodromic elements, then their fixed points are either disjoint or the same.*

Corollary 0.3. *If the limit set ΛG contains at least 3 points, then ΛG contains infinitely many points.*

Execise 0.4. *Let h, k be two loxodromic elements with disjoint fixed points. Then for any sufficiently large $n > 0$, the subgroup $\Gamma = \langle h^n, k^n \rangle$ is a free group of rank 2. Moreover, the orbital map*

$$g \in \Gamma \longmapsto go \in X$$

is a quasi-isometric embedding. (Tips: Use (C, D)-chain to obtain a quasi-geodesic for every word over $\{g^{\pm n}, h^{\pm n}\}$.)

Corollary 0.5. *A subgroup of a hyperbolic group is either virtually cyclic or contains a free group of rank 2.*