EXERCISE SHEET #5

Given a point $z \in X$ and a subset A, we denote by $\pi_A(z)$ the set of coarse projection points of z to A. Precisely, choose ϵ to be a very small positive number, say $\epsilon = 10^{-3}$ in what follows. Define

$$\pi_A(z) = \{a \in A : d(z,a) - d(z,A) \leq \epsilon\},$$

which is always non-empty.

Execise 0.1. *Let (X,d) be a δ-hyperbolic space. Let γ be a geodesic. Prove that there exists a constant $C > 0$ depending only on δ such that*

$$diam(\pi_\gamma(B)) \leq C$$

where B is a metric ball disjoint with γ.

Remark. The same conclusion holds for quasi-geodesics γ. Also one corollary is that the projection $\pi_\gamma(z)$ of a point z to a geodesic γ has uniformly bounded diameter.

Execise 0.2. *Let $f : X \to Y$ be a quasi-isometry between two δ-hyperbolic spaces. Let γ be a geodesic. Prove that there exists a constant $C > 0$ depending only on δ and the quasi-isometry constant of f such that for any $o \in X$, we have $f(\pi_\gamma(o))$ and $\pi_{f\gamma}(f(o))$ have a Hausdorff distance at most C:*

$$f(\pi_\gamma(o)) \subset N_C(\pi_{f\gamma}(f(o)))$$

and

$$\pi_{f\gamma}(f(o)) \subset N_C(f(\pi_\gamma(o))).$$

Remark. This result says that the projection π_* almost commutes with the quasi-isometry f (this does so, when f is isometry).