

Erratum: Gross–Zagier Formula On Shimura Curves

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1 Erratum

1. (p. 3, line 23) Change

“It is a regular scheme over F , locally noetherian but not of finite type”

to

“If $\#\Sigma > 1$, it is a regular scheme over F , locally noetherian but not of finite type.”

2. (p. 8, line 10) Change the equation

$$\Sigma(A, \chi) = \left\{ \text{places } v \text{ of } F : \epsilon\left(\frac{1}{2}, \pi_{A,v}, \chi_v\right) \neq \chi_v(-1)\eta_v(-1) \right\}$$

to

$$\Sigma(A, \chi) = \left\{ \text{places } v \text{ of } F : \epsilon\left(\frac{1}{2}, \pi_{A,v}, \chi_v\right) \neq \chi_v(-1) \right\}.$$

See also the correction for (p. 10, Thm 1.3) below.

3. (p. 8, line -8) Change

“for all places v ”

to

“for all non-archimedean places v .”

4. (p. 8, line -15) Change

“Fix Haar measures dt_v on E_v^\times/F_v^\times such that the product measure over all v gives the Tamagawa measure on $E_{\mathbb{A}}^\times/\mathbb{A}^\times$ ”

to

“Fix Haar measures dt_v on E_v^\times/F_v^\times such that the product measure over all v gives $\text{vol}(E^\times \backslash E_{\mathbb{A}}^\times/\mathbb{A}^\times) = 2L(1, \eta)$.”

See also (p. 23, line -2).

5. (p. 10, Thm 1.3) Change the equation

$$\epsilon\left(\frac{1}{2}, \pi, \chi\right) = \chi(-1)\eta(-1)\epsilon(B)$$

to

$$\epsilon\left(\frac{1}{2}, \pi, \chi\right) = \chi(-1)\epsilon(B).$$

In the literature, there are two conventional ways to define the root number $\epsilon(\frac{1}{2}, \pi, \chi)$, by viewing the L-function as either the base change L-function over E (twisted by χ) or the Rankin–Selberg L-function over F . The ratio of these two definitions are $\eta(-1)$. This paper takes the first convention, as written right before the theorem.

6. (p. 13, line 3) The formula

$$\theta(g, (h_1, h_2), \Phi) = \sum_{u \in F^\times} r(g, (h_1, h_2)) \Phi(x, u)$$

should be

$$\theta(g, (h_1, h_2), \Phi) = \sum_{u \in F^\times} \sum_{x \in B} r(g, (h_1, h_2)) \Phi(x, u).$$

7. (p. 13, line -9) In the summation, $\Phi(x)$ should be $\Phi(x_1, u)$.
8. (p. 15, line 4) In the formula, $\Phi(x, aq(x)^{-1})$ should be $\phi(x, aq(x)^{-1})$.
9. (p. 17, line -6) “ v nonsplit” should be “ v nonsplit”. This type of typo is very common in the book due to a misoperation on the latex file. Namely, in many places there should be a space before the word “nonsplit”. For example, in (p. 22, line 6), “ornonsplit” should be “or nonsplit”; in (p. 22, line 15), “isnonsplit” should be “is nonsplit”.
10. (p. 23, line -2) Remove the sentence

“All of them are Tamagawa measures.”

Note that the global measure on E^1 is not the Tamagawa measure due to the extra normalizing factor $L(1, \eta)$.

11. (p. 29, line -11) Change

$$\{(x_i, x_j)\}_{1 \leq i, j \leq \dim V}$$

to

$$\{\langle x_i, x_j \rangle\}_{1 \leq i, j \leq \dim V}.$$

12. (p. 31, line -2) Change

$$\mathcal{S}(V_v \times F_v)$$

to

$$\mathcal{S}(V_v \times F_v^\times).$$

13. (p. 33, line -9) Remove the sentence

“The Tamagawa number of $\mathrm{SO}(V)$ in the above cases are respectively $2L(1, \eta), 2, 2$.”

See also (p. 23, line -2).

14. (p. 40, line -5) Change

$$\int_{Z(F_v) \backslash T(F_v)} \chi(t) dt$$

to

$$\int_{Z(F_v) \backslash T(F_v)} \chi_v(t) dt.$$

15. (p. 43, line -9) Change

“Here the factor $4L(1, \eta)^2$ comes from the Tamagawa measure”

to

“Here the factor $4L(1, \eta)^2$ comes from the measure.”

See also (p. 23, line -2).

16. (p. 59, line 9) See (p. 3, line 23).

17. (p. 63, line 3) Change “ $f(UxU) = x$ ” to “ $f(UxU) = f(x)$ ”.

18. (p. 64, line 7) Change the second “ $H^{1,0}(X_{U,\tau})$ ” to “ $H^{0,1}(X_{U,\tau})$ ”.

19. (p. 68, line 9) Change

$$\text{Pic}(X_U \times X_U) \longrightarrow \text{Hom}^0(J_U, J_U^\vee)$$

to

$$\text{Pic}(X_U \times X_U) \longrightarrow \text{Hom}(J_U, J_U^\vee).$$

20. (p. 69, line -1) The definition should be

$$P_v(T, M) := \det_{M_\ell}(1 - \text{Frob}(v)|V_\ell(A)^{I_v})$$

to

$$P_v(T, M) := \det_{M_\ell}(1 - \text{Frob}(v)T|V_\ell(A)^{I_v}).$$

21. (p. 72, line 3, line 4) Change $\mathcal{A}(\mathbb{B})$ to $\mathcal{A}(\mathbb{B}^\times)$.

22. (p. 75, line 12) The last X_τ should be $X_{U,\tau}$.

23. (p. 77, line -4) Change the equation

$$L_v(s, A, M) = L(s, \pi_{A,v})$$

to

$$L_v(s, A, M) = L\left(s - \frac{1}{2}, \pi_{A,v}\right).$$

24. (p. 78, line 18) The isomorphism should be

$$J_U \simeq \bigoplus_{\pi \in \mathcal{A}(\mathbb{B}^\times, \mathbb{Q})} \tilde{\pi}^U \otimes_{\text{End}(\pi)} A_\pi.$$

25. (p. 82, line -2) Change

$$P_\chi(f_1) = \int_{\text{Gal}(\bar{E}/E)} f_1(P^\tau) \otimes_M \chi(\tau) d\tau$$

to

$$P_\chi(f_1) = \int_{\text{Gal}(E^{\text{ab}}/E)} f_1(P^\tau) \otimes_M \chi(\tau) d\tau.$$

26. (p. 84, line -8) Remove “an”.

27. (p. 84, line -6) Change the first “ $H^{1,0}(X_{U,\tau})^\vee$ ” to “ $H^{1,0}(X_{U,\tau})$ ”.

28. (p. 88, line 11) Change

$$\text{Hom}^0(J_U, J_U)$$

to

$$\text{Hom}^0(J_U, J_U^\vee).$$

29. (p. 91, line 1) Change “ $f_1 \in \pi_{A^v}^U$ ” to “ $f_2 \in \pi_{A^v}^U$ ”.
30. (p. 93, line -10) The special cycle $Z(x)_U$ should be defined as the push-forward (instead of the image) of X_{U_x} under the morphism

$$(\pi_{U_x, U}, \pi_{U_x, U} \circ \mathbf{T}_x) : X_{U_x} \longrightarrow X_U \times X_U.$$

This changes the definition by a constant multiple. The formula at (p. 93, line -5) still holds under this correction. This change affects the proof of Lemma 3.18 in page 96, but the lemma still holds. The change does not affect other parts of the paper. See §2 at the end of this note for more details on this change.

31. (p. 95, line -11) Change “The whole normalizing factor is always 2 if $F = \mathbb{Q}$ ” to “The whole normalizing factor is always $1/2$ if $F = \mathbb{Q}$ ”.
32. (p. 97, line 10) Change “tha” to “that”.
33. (p. 98, line 6) Change

$$C_0^\infty(\mathrm{GL}_2(F) \backslash \mathrm{GL}_2(\mathbb{A}), \chi|_{\mathbb{A}^\times})$$

to

$$C_0^\infty(\mathrm{GL}_2(F) \backslash \mathrm{GL}_2(\mathbb{A}), \chi|_{\mathbb{A}^\times}).$$

34. (p. 98, line 17) $[h] = T(h)P$ should be $[h] = T_h P$.

35. (p. 105, line 15) Change

$$\Sigma = \{v : \epsilon(\frac{1}{2}, \pi_v, \chi_v) \neq \chi_v \eta_v(-1)\}$$

to

$$\Sigma = \{v : \epsilon(\frac{1}{2}, \pi_v, \chi_v) \neq \chi_v(-1)\}.$$

36. (p. 109, line 10) The goal of this item is not to state a mistake but to explain the expression

$$\theta(g, \phi)_K = \sum_{u \in \mu_K^2 \backslash F^\times} r(g)\phi(0, u) + w_K \sum_{(x, u) \in \mu_K \backslash ((V - \{0\}) \times F^\times)} r(g)\phi(x, u).$$

Then it suffices to check

$$\sum_{u \in \mu_K^2 \backslash F^\times} \sum_{x \in V'} r(g)\phi(x, u) = w_K \sum_{(x, u) \in \mu_K \backslash (V' \times F^\times)} r(g)\phi(x, u).$$

Here we denote $V' = V - \{0\}$.

Let A be a subset of V' such that the induced map $A \rightarrow \omega_K \backslash V'$ is bijective, and let B be a subset of F^\times such that the induced map $B \rightarrow \mu_K^2 \backslash F^\times$ is bijective. Here $\omega_K = \{\pm 1\} \cap K^\times$, and thus $w_K = |\omega_K|$. One checks that the natural map

$$A \times B \longrightarrow \mu_K \backslash (V' \times F^\times)$$

is bijective. As a consequence, we have

$$\sum_{u \in \mu_K^2 \backslash F^\times} \sum_{x \in \omega_K \backslash V'} r(g)\phi(x, u) = \sum_{(x, u) \in \mu_K \backslash (V' \times F^\times)} r(g)\phi(x, u).$$

This gives the desired equality.

37. (p. 112, lines 5-7) The mistake is similar to that in (p. 93, line -10). The special cycle $Z(y)_{K'}$ is not a curve in $M'_{K'}$, but the push-forward of a curve. The push-forward (instead of the image) of $Z(y)_{K'}$ under the natural map $M'_{K'} \rightarrow M_K$ is $Z(y)_U$.

38. (p. 112, line 9) Change “different” to “the same”.

39. (p. 114, line -2) Change

$$\mathbb{V}_f(b) = \{x \in \mathbb{V} : q(x) = b\}$$

to

$$\mathbb{V}_f(b) = \{x \in \mathbb{V}_f : q(x) = b\}.$$

40. (p. 115, line -9 and line -7; p. 116, line 4) Change

$$M'_{K'}$$

to

$$M'_{K^h}$$

41. (p. 116, line 16) Change

$$\ell(Z(g, \Phi)_U |_{M_K^{\circ}})$$

to

$$\ell(Z(g, \phi)_U |_{M_K^{\circ}}).$$

42. (p. 116, line 20) Change $\phi = \phi_f \times \phi_{\infty}$ to $\phi = \phi_f \otimes \phi_{\infty}$.

43. (p. 117, line 2) Change

$$M_K - M_K^{\circ} = (X_U \times \{\text{cusps}\}) \cup (X_U \times \{\text{cusps}\})$$

to

$$M_K - M_K^{\circ} = (X_U \times \{\text{cusps}\}) \cup (\{\text{cusps}\} \times X_U).$$

44. (p. 120, line -4) Change \mathbb{B}_f^{\times} to \mathbb{B}^{\times} .

45. (p. 126, line -5) Change

$$\phi \in \overline{\mathcal{S}}(\mathbb{A} \times \mathbb{A}^{\times})$$

to

$$\phi \in \overline{\mathcal{S}}(\mathbb{V} \times \mathbb{A}^{\times}).$$

46. (p. 137, line 8) Change

“The first identity is just the result for the Tamagawa number of”

to

“The first identity is just the result for the measure of”

See also (p. 23, line -2).

47. (p. 140, Proposition 4.11) In the main formula, there should be a negative sign in the second summation of the right-hand side. Hence, the correct formula is

$$\Delta^* Z(g, \phi)_U = \sum_{u \in \mathbb{Q}^{\times}} \theta(g, u, \phi^0) C(g, u, \phi_0)_U - \sum_{u \in \mathbb{Q}^{\times}} W_0(g, u, \phi) L_U + D(g, \phi)_U.$$

Accordingly, in the proof of the proposition, the expression of S should be negated. This proposition is used in the proof of Theorem 4.15, but the formula in the second paragraph has the corrected form.

48. (p. 184, line -10) Change “§6.2” to “§6.3”.

49. (p. 185, line 2) $I(s, g, \phi)_U$ cannot be written that way until it is explained below. At present, it should be

$$I(s, g, \phi)_U = \sum_{u \in \mu_v^2 \backslash F^\times} \sum_{\gamma \in P^1(F) \backslash \mathrm{SL}_2(F)} \delta(\gamma g)^s \sum_{x_1 \in E} r(\gamma g) \phi(x_1, u).$$

50. (p. 186, line -3) Change “ $\prod_v \gamma_{u,v} = 1$ ” to “ $\prod_v \gamma_{u,v} = -1$ ”.

51. (p. 187, line 4) Change

$$W_0^\circ(0, g, \phi) = r(g) \phi(0, u)$$

to

$$W_0^\circ(0, g, \phi_2) = r(g) \phi_2(0, u).$$

52. (p. 191, line -2) Change “ $y = y_1 + y_2 \in V_v - V_{1v}$ ” to “ $y = y_1 + y_2 \in V_v$ ”.

53. (p. 196, line -2) Change “Proposition 6.6” to “Lemma 6.6”.

54. (p. 198, line -11) Change

$$\mathcal{P}r(f)_\psi(g) = W^{(2)}(g_\infty) \mathcal{P}r(f)_\psi(g)$$

to

$$\mathcal{P}r(f)_\psi(g) = W^{(2)}(g_\infty) \mathcal{P}r(f)_{\psi,f}(g).$$

55. (p. 200, Proof of Lemma 6.13) The quotient $F^\times \backslash \mathbb{A}^\times / F_\tau^\times Q$ is not finite if $F \neq \mathbb{Q}$, but $F^\times \backslash \mathbb{A}^\times / F_\infty^\times Q$ is always finite. We should replace $F^\times \backslash \mathbb{A}^\times / F_\tau^\times Q$ by $F^\times \backslash \mathbb{A}^\times / F_\infty^\times Q$ in the proof.

56. (p. 219, line 8) Delete \sum_v before $i_{\bar{v}}(t_1 x, t_2)$.

57. (p. 225, line 14; p. 227, line 5) There is a negative sign missing on the right-hand side of the formula. The coefficient 2 before the averaged integral should be -2 .

58. (p. 225, line 17) Change

$$\sum_{v \nmid \infty \text{ nonsplit}} \bar{\mathcal{K}}_\phi^{(v)}(g, (t_1, t_2))$$

to

$$\sum_{v \nmid \infty \text{ nonsplit}} \mathcal{K}_\phi^{(v)}(g, (t_1, t_2)).$$

59. (p. 228, line 7) “Langland” should be “Langlands”.

60. (p. 228, line 10) Change

$$P_\chi(f_i) = \int_{T(F) \backslash T(\mathbb{A}) / Z(\mathbb{A})} f_i(t) dt$$

to

$$P_\chi(f_i) = \int_{T(F) \backslash T(\mathbb{A}) / Z(\mathbb{A})} f_i(t) \chi(t) dt.$$

61. (p. 228, line 14) Change

$$\mathrm{Hom}_{E_{\mathbb{A}^\times}}(\pi(v) \otimes \chi, \mathbb{C})$$

to

$$\mathrm{Hom}_{E_{\mathbb{A}^\times}}(\pi(v) \otimes \chi, \mathbb{C}).$$

62. (p. 231, line 9; p. 232, line 7) The Green's function $\widetilde{\lim}_{s \rightarrow 0} g_s$ is not admissible in the sense of §7.1.15, since it does not satisfy the vanishing condition on (p. 212, line -6). Then there should be some extra terms in the computation of the archimedean local heights. However, the contributions of these extra terms are actually zero under Assumption 5.4 in page 174. Therefore, this mistake does not affect the main results. See clarification and correction of this issue in the preprint

X. Yuan, Modular Heights of Quaternionic Shimura Curves, arXiv:2205.13995.

See especially §4.1, Lemma 4.1(2), Remark 4.3, and Remark 4.5 of the paper.

63. (p. 233, line 15) The formula

$$\int_1^\infty \frac{1}{t(1-\lambda t)^{s+1}} dt = 2Q_s(1-2\lambda) + O(|\lambda|^{-s-2})$$

should be

$$\int_1^\infty \frac{1}{t(1-\lambda t)^{s+1}} dt = 2 \frac{\Gamma(2s+2)}{\Gamma(s+1)\Gamma(s+2)} Q_s(1-2\lambda) + O(|\lambda|^{-s-2}).$$

This follows from the calculation in p. 304 of Gross–Zagier's paper and especially the formula in line 12 of the page.

64. (p. 234, line 3) X_K should be X_U .

2 Definition of the special cycle $Z(x)_U$

In (p. 93, §3.4.3), there are two descriptions (or definitions) of $Z(x)_U$. These two are not equivalent, and one of them is incorrect. Recall that the descriptions are:

- (a) $Z(x)_U$ can be defined as the **image** of the morphism

$$(\pi_{U_x, U}, \pi_{U_x, U} \circ \mathbb{T}_x) : X_{U_x} \longrightarrow X_U \times X_U.$$

Here $U_x = U \cap xUx^{-1}$ is an open and compact subgroup of \mathbb{B}_f^\times . It is an algebraic correspondence on X_U , and also viewed as an element of $\text{Pic}(X_U \times X_U)$.

- (b) In terms of complex uniformization above, the push-forward gives

$$(Z(x)_U)_* : [z, \beta]_U \longmapsto \sum_{y \in U_x U / U} [z, \beta y]_U.$$

Description (b) is the correct one, and most of the book, including all computation of height series, is based on this definition. Definition (a) is not correct, but the correct one is:

- (a') $Z(x)_U$ can be defined as the **push-forward** of the morphism

$$\Psi = (\pi_{U_x, U}, \pi_{U_x, U} \circ \mathbb{T}_x) : X_{U_x} \longrightarrow X_U \times X_U.$$

Here $U_x = U \cap xUx^{-1}$ is an open and compact subgroup of \mathbb{B}_f^\times . It is an algebraic correspondence on X_U , and also viewed as an element of $\text{Pic}(X_U \times X_U)$.

The image $\Psi(X_{U_x})$ is an integral curve in $X_U \times X_U$. The push-forward is just

$$\Psi_*(X_{U_x}) = \deg(X_{U_x} \rightarrow \Psi(X_{U_x})) \Psi(X_{U_x}).$$

Note that the degree is computed in Lemma 2.4, and it is not necessarily 1.

Both (a') and (b) are the correct descriptions for this book. This issue affects the proof of Lemma 3.18 in page 96, but the lemma is still correct by a revised proof. The other parts of the book are based on either (a') or (b), so they are not affected.

In the following, we check that (a') implies (b), and then correct the proof of Lemma 3.18 in page 96. We always assume that $Z(x)_U$ is defined by (a').

2.1 The action on points

Let $Z(x)_U$ be as in (a'), and we will that $Z(x)_U$ always satisfies (b). Namely, we will prove the following result.

Theorem 2.1. *For any point $[z, \beta]_U \in X_{U, \tau}(\mathcal{C})$,*

$$(Z(x)_U)_* : [z, \beta]_U \mapsto \sum_{y \in UxU/U} [z, \beta y]_U.$$

The result also holds for cusps, but we will do not need it. To prove the theorem, we set up some notations. Denote

$$\bar{U} := U/(U \cap F^\times) = U/\mu_U.$$

For any open compact subgroups U and U' of \mathbb{B}_f^\times , denote

$$[U : U'] = \text{vol}(U)/\text{vol}(U'), \quad [\bar{U} : \bar{U}'] = \text{vol}(\bar{U})/\text{vol}(\bar{U}').$$

Note that these give the usual indices if U' is contained in U . We have the following easy result.

Lemma 2.2. *For open compact subgroups $U' \subset U$ of \mathbb{B}_f^\times , the natural map*

$$U/U' \longrightarrow \bar{U}/\bar{U}'$$

is surjective, and every element of \bar{U}/\bar{U}' has exactly $[\mu_U : \mu_{U'}]$ preimages.

Proof. For $u \in U$, the preimage of $u\bar{U}'$ is just $u\mu_U U'/U'$. □

Let U and U' be open compact subgroups of \mathbb{B}_f^\times such that $U' \subset xUx^{-1}$ for some $x \in \mathbb{B}_f^\times$. Denote by

$$\pi_{U', U, x} = \pi_{U', U} \circ T_x : X_{U'} \longrightarrow X_U$$

the morphism given by right multiplication by x . We have the following result.

Lemma 2.3. *For any $[z, \beta]_U \in X_{U, \tau}(\mathcal{C})$, the divisor*

$$\pi_{U', U, x}^*([z, \beta]_U) = \sum_{\gamma \in (x\bar{U}x^{-1})/\bar{U}'} [z, \beta x^{-1}\gamma]_{U'}.$$

As a consequence,

$$\deg(\pi_{U', U, x}) = [(x\bar{U}x^{-1}) : \bar{U}'] = [\bar{U} : \bar{U}']$$

Proof. We first consider the case where $[z, \beta]_U$ is non-cusp, non-CM, and unramified under $\pi_{U', U, x}$. The preimage of $[z, \beta]_U$, as a set, is given by the image of $[z, F^\times \beta Ux^{-1}]$ in $X_{U', \tau}(\mathcal{C})$. In this case, we simply have

$$\pi_{U', U, x}^*([z, \beta]_U) = \sum_{\gamma \in F^\times \backslash F^\times \beta Ux^{-1}/U'} [z, \gamma]_{U'} = \sum_{\gamma \in \beta \bar{U}x^{-1}/\bar{U}'} [z, \gamma]_{U'} = \sum_{\gamma \in (x\bar{U}x^{-1})/\bar{U}'} [z, \beta x^{-1}\gamma]_{U'}.$$

There are two ways to extend the formula to all $[z, \beta]_U$. The pleasant way is to approximate $[z, \beta]_U$ by unramified and non-points $[z', \beta]_U$ with $z' \rightarrow z$ in \mathcal{H}^\pm . This extends the formula to all z (including the cusps).

Alternatively, we can also compute the ramification indices directly. This goes through very tedious calculations, but it eventually gives the result. □

Denote

$$\Psi = \Psi_{U,x} = (\pi_{U_x,U}, \pi_{U_x,U,x}), \quad \pi_1 = \pi_{U_x,U}, \quad \pi_2 = \pi_{U_x,U,x}.$$

So the morphism

$$(\pi_{U_x,U}, \pi_{U_x,U,x}) : X_{U_x} \longrightarrow X_U \times X_U.$$

is written as

$$\Psi = (\pi_1, \pi_2) : X_{U_x} \longrightarrow X_U \times X_U.$$

Lemma 2.4. *The degree*

$$\deg(X_{U_x} \rightarrow \Psi(X_{U_x})) = [\overline{U} \cap (x\overline{U}x^{-1}) : \overline{U}_x].$$

It is not necessarily 1.

Proof. Let $[z, \beta]_{U_x}$ with $\beta \in \mathbb{B}_f^\times$ be a point of $X_{U_x, \tau}(\mathcal{C})$, and assume that z is neither a cusp nor a CM point. Any point of $\Psi^{-1}(\Psi([z, \beta]_{U_x}))$ is represented by a point of the form $[z, \beta']_{U_x}$ with $\beta' \in \mathbb{B}_f^\times$ satisfying

$$[z, \beta']_U = [z, \beta]_U, \quad [z, \beta'x]_U = [z, \beta x]_U.$$

The conditions is equivalent to

$$\beta' \in F^\times \beta U, \quad \beta'x \in F^\times \beta x U,$$

and thus equivalent to

$$\beta' \in (F^\times \beta U) \cap (F^\times \beta x U x^{-1}).$$

Then the number of such $[z, \beta']_{U_x}$ is equal to

$$\left| F^\times \setminus \left((F^\times \beta U) \cap (F^\times \beta x U x^{-1}) \right) / U_x \right| = \left| \left((F^\times U) \cap (F^\times x U x^{-1}) \right) / (F^\times U_x) \right| = [\overline{U} \cap (x\overline{U}x^{-1}) : \overline{U}_x].$$

□

Now we prove Theorem 2.1.

Proof of Theorem 2.1. We have

$$\Psi_*(X_{U_x})([z, \beta]_U) = (\pi_2)_* \pi_1^*([z, \beta]_U) = \sum_{\gamma \in \overline{U}/\overline{U}_x} (\pi_2)_*([z, \beta\gamma]_{U_x}) = \sum_{\gamma \in \overline{U}/\overline{U}_x} [z, \beta\gamma x]_U.$$

Here the first equality follows from a basic property of correspondence, and the second equality follows from Lemma 2.3. By Lemma 2.2, we further have

$$\Psi_*(X_{U_x})([z, \beta]_U) = \frac{1}{[\mu_U : \mu_{U_x}]} \sum_{\gamma \in U/U_x} [z, \beta\gamma x]_U.$$

By definition,

$$\mu_{U_x} = F^\times \cap U \cap (xUx^{-1}) = \mu_U \cap (xUx^{-1}) = x((x^{-1}\mu_U x) \cap U)x^{-1} = x(\mu_U \cap U)x^{-1} = x\mu_U x^{-1} = \mu_U.$$

It follows that

$$\Psi_*(X_{U_x})([z, \beta]_U) = \sum_{\gamma \in U/U_x} [z, \beta\gamma x]_U = \sum_{y \in UxU/U} [z, \beta y]_U.$$

Here the last inequality comes from the bijection

$$U/U_x \longrightarrow UxU/U, \quad \gamma U_x \longmapsto \gamma x U.$$

□

2.2 Pull-back of $Z(x)_U$

(page 96, line 9) $h_1 U' h_1^{-1} \subset U$ and $h_2 U' h_2^{-1} \subset U$ should be $h_1^{-1} U' h_1 \subset U$ and $h_2^{-1} U' h_2 \subset U$.

(page 97, line 7) The formula

$$\Pi^* Z(x)_U = \sum_{h_1^{-1} x' h_2 \in (F^\times U') \setminus F^\times U x U' / U'}$$

is wrong, and requires a coefficient. The correct one is the following.

Lemma 2.5. *We have*

$$\Pi^* Z(x)_U = \frac{1}{[\mu_U : \mu_{U'}]} \sum_{x' \in U' \setminus (h_1 U x U h_2^{-1}) / U'}$$

Proof. Write $\Pi = (\pi_1, \pi_2)$, where $\pi_i : X_{U'} \rightarrow X_U$ is given by right multiplication by h_i . Take a point $[z, \beta]_{U'}$ (with $\beta \in \mathbb{B}_f^\times$) of $X_{U', \tau}(\mathcal{C})$, and assume that z is not a CM point. Apply Lemma 3.1.2 of the book. We have

$$(\Pi^* Z(x)_U)([z, \beta]_{U'}) = \pi_2^*(Z(x)_U(\pi_1([z, \beta]_{U'}))).$$

Note that the result also holds for $\pi_1 \neq \pi_2$ by the original reference [Li, Proposition 1.13]. Thus we have

$$(\Pi^* Z(x)_U)([z, \beta]_{U'}) = \pi_2^*(Z(x)_U([z, \beta h_1]_U)) = \sum_{y \in U x U / U} \pi_2^*([z, \beta h_1 y]_U) = \sum_{y \in U x U / U} \sum_{\gamma \in (h_2 U h_2^{-1}) / U'} [z, \beta h_1 y h_2^{-1} \gamma]_{U'}.$$

Here the second equality follows from Theorem 2.1, and the third equality follows from Lemma 2.3.

By Lemma 2.2, we further transfer the above expression to

$$\begin{aligned} (\Pi^* Z(x)_U)([z, \beta]_{U'}) &= \frac{1}{[\mu_{h_2 U h_2^{-1}} : \mu_{U'}]} \sum_{y \in U x U / U} \sum_{\gamma \in (h_2 U h_2^{-1}) / U'} [z, \beta h_1 y h_2^{-1} \gamma]_{U'} \\ &= \frac{1}{[\mu_U : \mu_{U'}]} \sum_{w \in (h_2 U x U h_2^{-1}) / (h_2 U h_2^{-1})} \sum_{\gamma \in (h_2 U h_2^{-1}) / U'} [z, \beta h_1 h_2^{-1} w \gamma]_{U'} \\ &= \frac{1}{[\mu_U : \mu_{U'}]} \sum_{w \in (h_2 U x U h_2^{-1}) / U'} [z, \beta h_1 h_2^{-1} w]_{U'} \\ &= \frac{1}{[\mu_U : \mu_{U'}]} \sum_{v \in (h_1 U x U h_2^{-1}) / U'} [z, \beta v]_{U'} \\ &= \frac{1}{[\mu_U : \mu_{U'}]} \sum_{v \in U' \setminus (h_1 U x U h_2^{-1}) / U'} Z(v)_{U'}([z, \beta]_{U'}). \end{aligned}$$

Here the second equality is given by the substitution $y = h_2^{-1} w h_2$, the third equality is given a contraction of variables, and the fourth inequality is given by the substitution $w = h_2 h_1^{-1} v$. This implies

$$\Pi^* Z(x)_U = \frac{1}{[\mu_U : \mu_{U'}]} \sum_{v \in U' \setminus (h_1 U x U h_2^{-1}) / U'} Z(v)_{U'}.$$

□

(page 97, line 10) The formula $U \setminus F^\times U h_1^{-1} x' h_2 U / U = \mu_U \setminus F^\times h_1^{-1} x' h_2$ is also wrong.

(page 96, line 11) The formula

$$\Pi^* Z_a(\Phi)_U = \frac{1}{[\mu_U^2 : \mu_{U'}^2]} Z_a(r(h_1, h_2)\Phi)_{U'}$$

is still correct. The correction of the proof after (page 96, line 3) should be as follows.

By Lemma 2.5, we have

$$\begin{aligned}
\Pi^* Z_a(\Phi)_U &= w_U \sum_{x \in U \setminus \mathbb{B}_f^\times / U} \bar{\Phi}(x, aq(x)^{-1}) \Pi^* Z(x)_U \\
&= \frac{w_U}{[\mu_U : \mu_{U'}]} \sum_{x \in U \setminus \mathbb{B}_f^\times / U} \bar{\Phi}(x, aq(x)^{-1}) \sum_{x' \in U' \setminus (h_1 U x U h_2^{-1}) / U'} Z(x')_{U'} \\
&= \frac{w_U}{[\mu_U : \mu_{U'}]} \sum_{x' \in U' \setminus \mathbb{B}_f^\times / U'} \bar{\Phi}(h_1^{-1} x' h_2, aq(h_1^{-1} x' h_2)^{-1}) Z(x')_{U'} \\
&= \frac{w_U}{[\mu_U : \mu_{U'}]} \sum_{x' \in U' \setminus \mathbb{B}_f^\times / U'} r(h_1, h_2) \bar{\Phi}(x', aq(x')^{-1}) Z(x')_{U'} \\
&= \frac{w_U / w_{U'}}{[\mu_U : \mu_{U'}]} Z_a(r(h_1, h_2) \Phi)_{U'}.
\end{aligned}$$

Note that $[\mu_U : \mu_{U'}^2] = 2^{[F:\mathbb{Q}]-1} w_U$. The above becomes

$$\Pi^* Z_a(\Phi)_U = \frac{1}{[\mu_{U'}^2 : \mu_{U'}]} Z_a(r(h_1, h_2) \Phi)_{U'}.$$

This finishes the correction.

We remark that to avoid complication by double cosets, it will be more convenient to prove the above by verifying

$$\Pi^* Z_a(\Phi)_U([z, \beta]_{U'}) = \frac{1}{[\mu_{U'}^2 : \mu_{U'}]} Z_a(r(h_1, h_2) \Phi)_{U'}([z, \beta]_{U'})$$

as in the proof of Lemma 2.5.

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