Topics in Analysis of Many Particle Systems

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December 15, 2020

The goal of this course is to introduce the recent very popular research topic in the flavor of large N limit. This type of problems tries to bridge the connections between microscopic descriptions, usually given by (discrete) ODE or SDE models, to macroscopic descriptions, usually given by (continuous) PDE models. We will present some mathematical tools and concepts for the rigorous derivation and study of nonlinear partial differential equations (PDE's) arising from many-particle limits: (McKean-)Vlasov type equations, the vorticity formulation of the 2D incompressible Euler/Navier-Stokes equations, Boltzmann collision equations, nonlinear diffusion equations, quantum Hartree equations...

Depending on time and interest it will include part or all of the following topics: the Liouville/Master equations of N-particle systems, the notion of empirical measures, the BBGKY hierarchy, the Hewitt-Savage theorem, the Dobrushin's stability estimate, the coupling method, the concepts of chaos and entropic chaos, the recent progresses on the mean-field limit, in particular, the relative entropy/modulated potential energy/modulated free energy methods as introduced in [6, 7, 13, 8, 9, 10].

The instructor expects to spend half of the semester on the classical and recent results on Mean Field Limit problems and another half on the related (similar) large N limit problem (for instance in the quantum setting) and the analysis of Mean-Field PDEs (for instance Aggregation-Diffusion PDEs and some Kinetic equations).

Pre-requisites: Basics in measure theory, real and functional analysis, partial differential equations and probability.
Textbook: We will not use any textbook. But most materials will be based on the notes [4, 5] and recent articles [6, 7, 13, 8, 9, 10].
Time: TBA.
Location: TBA
Examination: TBA. (Presentation/Oral exam/Final project)

References

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