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习题 3.1

1. $\int \sqrt{1+2x}dx = \frac{1}{2}\int \sqrt{1+2x}d(1+2x) = \frac{1}{3}(1+2x)^{3/2} + C.$
2. $\int \frac{3x}{\left(x^2+1\right)^2}dx = \frac{1}{2}\int \frac{3}{\left(x^2+1\right)^2}d\left(x^2+1\right) = -\frac{3}{2\left(x^2+1\right)} + C.$
3. $\int x\sqrt{2x^2+7}dx = \frac{1}{4}\int \sqrt{2x^2+7}d\left(2x^2+7\right) = \frac{1}{6}\left(2x^2+7\right)^{3/2} + C.$
4. $\int \left(2x^{3/2}+1\right)^{2/3}\sqrt{x}dx = \frac{2}{3}\int \left(2x^{3/2}+1\right)^{2/3}dx^{3/2}$
$= \frac{2}{3}\,\mathrm{g}_2^1\int \left(2x^{3/2}+1\right)^{2/3}d\left(2x^{3/2}+1\right) = \frac{1}{5}\left(2x^{3/2}+1\right)^{5/3} + C.$
5. $\int \frac{e^{1/x}}{x^2}dx = -\int e^{1/x}d(1/x) = -e^{1/x} + C.$
6. $\int \frac{dx}{(2-x)^{100}} = -\int \frac{d(2-x)}{(2-x)^{100}} = \frac{1}{99(2-x)^{99}} + C.$
7. $\int \frac{dx}{3+5x^2} = \frac{1}{3}\int \frac{dx}{1+[(5/3)x]^2} = \frac{1}{3}\sqrt{\frac{3}{5}}\int \frac{d\sqrt{5/3}x}{1+[\sqrt{5/3}x]^2} = \frac{1}{\sqrt{15}}\arctan\sqrt{\frac{5}{3}}x + C.$
8. $\int \frac{dx}{\sqrt{7-3x^2}} = \int \frac{dx}{\sqrt{7}\sqrt{1-3/7x^2}} = \frac{1}{\sqrt{7}}\,\mathrm{g}\sqrt{\frac{7}{3}}\int \frac{d\sqrt{3/7}x}{\sqrt{7}\sqrt{1-\sqrt{3/7}x^2}} = \frac{1}{\sqrt{3}}\arcsin\sqrt{\frac{3}{7}}x + C$ (1)
9. $\int \frac{dx}{\sqrt{x}(1+x)} = 2\int \frac{d\sqrt{x}}{(1+x)} = 2\arctan\sqrt{x} + C.$
10. $\int \frac{e^x}{2+e^{2x}}dx = \int \frac{1}{2+\left(e^x\right)^2}de^x = \frac{1}{\sqrt{2}}\arctan e^x + C$
11. $\int \frac{dx}{\sqrt{e^{-2x}-1}} = \int \frac{de^x}{\sqrt{1-\left(e^x\right)^2}} = \arcsin e^x + C.$
12. $\int \frac{dx}{e^x-e^{-x}} = \int \frac{de^x}{e^{2x}-1} = \int \frac{du}{(u-1)(u+1)} = \frac{1}{2}\int \left(\frac{1}{u-1} - \frac{1}{u+1}\right)du$
$= \frac{1}{2}\ln\frac{u-1}{u+1} + C = \frac{1}{2}\ln\frac{e^x-1}{e^x+1} + C.$
13. $\int \frac{\ln\ln x}{x\ln x}dx = \int \frac{\ln\ln x}{\ln x}d\ln x = \int \ln\ln x d\ln\ln x = \frac{1}{2}(\ln\ln x)^2 + C.$
14. $\int \frac{dx}{1+\cos x} = \int \frac{dx}{2\sin^2\frac{x}{2}} = \int \frac{d\frac{x}{2}}{\sin^2\frac{x}{2}} = -\cot^2\frac{x}{2} + C$

15. $\int \frac{dx}{1-\sin x} = \int \frac{d\left(x+\frac{\pi}{2}\right)}{1+\cos\left(x+\frac{\pi}{2}\right)} = -\cot^2\left(\frac{x}{2}+\frac{\pi}{4}\right)+C$

16. $\int \frac{x^{14}}{\left(x^5+1\right)^4}dx = \frac{1}{5}\int \frac{x^{10}}{\left(x^5+1\right)^4}dx^5 = \frac{1}{5}\int \frac{u^2}{(u+1)^4}du\left(u=x^5\right)$

$= \frac{1}{5}\int \frac{u^2-1+1}{(u+1)^4}du = \frac{1}{5}\int \frac{(v-1)^2}{v^4}dv(v=u+1)$

$= \frac{1}{5}\int \frac{v^2-2v+1}{v^4}dv = \frac{1}{5}\int \left(v^{-2}-2v^{-3}+v^{-4}\right)dv$

$= \frac{1}{5}\left(-v^{-1}+v^{-2}-\frac{1}{3}v^{-3}\right)+C = \frac{1}{5}\left(-\left(x^5+1\right)^{-1}+\left(x^5+1\right)^{-2}-\frac{1}{3}\left(x^5+1\right)^{-3}\right)+C$

17. $\int \frac{x^{2n-1}}{x^n-1}dx = \frac{1}{n}\int \frac{x^n}{x^n-1}dx^n = \frac{1}{n}\int \frac{u}{u-1}du\,(u=x^n)$

$= \frac{1}{n}\int \left(1+\frac{1}{u-1}\right)du = \frac{1}{n}(u+\ln|u-1|)+C = \frac{1}{n}\left(x^n+\ln|x^n-1|\right)+C.$

18. $\int \frac{dx}{x\left(x^5+2\right)} = \int \frac{x^4dx}{x^5\left(x^5+2\right)} = \frac{1}{5}\int \frac{du}{u(u+2)}\left(u=x^5\right)$ (2)

$= \frac{1}{5}\int \left(\frac{1}{u}-\frac{1}{u+2}\right)du = \frac{1}{10}(\ln|u|-\ln|u+2|)+C = \frac{1}{10}\ln\left|\frac{u}{u+2}\right|+C$

19. $\int \frac{\ln(x+1)-\ln x}{x(x+1)}dx = \int (\ln(x+1)-\ln x)\left(\frac{1}{x}-\frac{1}{x+1}\right)dx$

$= \int (\ln(x+1)-\ln x)d\left(\ln x-\ln(x+1)\right. = -\int (\ln(x+1)-\ln x)d(\ln(x+1)-\ln x)$

$= -\frac{1}{2}\ln^2\frac{x+1}{x}+C.$

20. $\int \frac{e^{\arctan x}+x\ln\left(1+x^2\right)}{1+x^2}dx = \int \frac{e^{\arctan x}}{1+x^2}dx+\int \frac{x\ln\left(1+x^2\right)}{1+x^2}dx$

$= \int e^{\arctan x}d\arctan x+\frac{1}{2}\int \ln\left(1+x^2\right)d\ln\left(1+x^2\right)$

$= e^{\arctan x}+\frac{1}{4}\ln^2\left(1+x^2\right)+C$

21. $\int \sin 2x \cos 2x dx = \frac{1}{2}\int \sin 2x d\sin 2x = \frac{1}{4}\sin^2 2x + C$

22. $\int \sin^2 \frac{x}{2}\cos\frac{x}{2}dx = 2\int \sin^2\frac{x}{2} d\sin\frac{x}{2} = \frac{2}{3}\sin^3\frac{x}{2} + C.$

23. $\int \sin 5x \sin 6x dx = \frac{1}{2}\int(\cos x - \cos 11x)dx = \frac{1}{2}\left(\sin x - \frac{1}{11}\sin 11x\right) + C.$

24. $\int \frac{2x-1}{\sqrt{1-x^2}}dx = \int \frac{2x}{\sqrt{1-x^2}}dx - \int \frac{1}{\sqrt{1-x^2}}dx$

$= -\int \frac{d\left(1-x^2\right)}{\sqrt{1-x^2}} - \arcsin x + C = -2\sqrt{1-x^2} - \arcsin x + C$

25 $\cdot \int \frac{x^3+x}{\sqrt{1-x^2}}dx = \int \frac{x^3}{\sqrt{1-x^2}}dx + \int \frac{x}{\sqrt{1-x^2}}dx$

$= \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}dx^2 - \frac{1}{2}\int \frac{d\left(1-x^2\right)}{\sqrt{1-x^2}}$

$= \frac{1}{2}\int \sqrt{1-x^2}d\left(1-x^2\right) + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx^2 - \sqrt{1-x^2}$

$= \frac{1}{3}\left(1-x^2\right)^{3/2} - 2\sqrt{1-x^2} + C$ (3)

26. $\int \frac{dx}{\left(a^2-x^2\right)^{3/2}}(a>0)$

$x = a\sin t, t \in (-\pi/2, \pi/2), dx = a\cos t dt$

$\left(a^2-x^2\right)^{3/2} = a^3\cos^3 t$

$\int \frac{dx}{\left(a^2-x^2\right)^{3/2}} = \int \frac{dt}{a^2\cos^2 t}dx = \frac{1}{a^2}\tan t + C$

$= \frac{1}{a^2}\frac{x/a}{\sqrt{1-(x/a)^2}} + C = \frac{x}{a^2\sqrt{a^2-x^2}} + C$

$x < 0$ 时, 令$x = -y, y > 0,$

$\int \frac{\sqrt{x^2-a^2}}{x}dx = \int \frac{\sqrt{y^2-a^2}}{y}dy = \sqrt{y^2-a^2} - a\arccos\frac{a}{y} + C$

$= \sqrt{x^2-a^2} - a\arccos\frac{a}{-x} + C = \sqrt{x^2-a^2} - \left(\pi - a\arccos\frac{a}{x}\right) + C$

$= \sqrt{x^2-a^2} + a\arccos\frac{a}{x} + C'.$

27. $\int \frac{\sqrt{x^2-a^2}}{x}dx(a>0)x>0$ 时, 令$x=a\sec t, t\in(0,\pi/2)$

$dx=a\tan t\sec tdt, \sqrt{x^2-a^2}=a\tan t$

$$\int \frac{\sqrt{x^2-a^2}}{x}dx=a\int \tan^2 tdt=a\int\left(\sec^2 t-1\right)dt=a(\tan t-t)+C$$

$$=a\left(\sqrt{\sec^2 t-1}-\arccos\frac{a}{x}\right)+C=a\left(\sqrt{\left(\frac{x}{a}\right)^2-1}-\arccos\frac{a}{x}\right)+C$$

$$=\sqrt{x^2-a^2}-a\arccos\frac{a}{x}+C$$

28. $$\int \frac{x^2}{\sqrt{a^2-x^2}}dx=-\int\sqrt{a^2-x^2}dx+a^2\int\frac{dx}{\sqrt{a^2-x^2}}$$

$$=-\frac{a^2}{2}\arcsin\frac{x}{a}-\frac{1}{2}x\sqrt{a^2-x^2}+a^2\arcsin\frac{x}{a}+C$$

$$=\frac{a^2}{2}\arcsin\frac{x}{a}-\frac{1}{2}x\sqrt{a^2-x^2}+C$$

29. $$\int\frac{dx}{\sqrt{1+e^{3x}}}=\int\frac{e^{-3x/2}dx}{\sqrt{1+e^{-3x}}}=-\frac{2}{3}\int\frac{de^{-3x/2}}{\sqrt{1+e^{-3x}}}=-\frac{2}{3}\ln\left(e^{-3x/2}+\sqrt{1+e^{-3x}}\right)+C$$

$$=-\frac{2}{3}\ln\left(1+\sqrt{1+e^{3x}}\right)+x+C=-\frac{2}{3}\ln\frac{\left(\sqrt{1+e^{3x}}\right.}{\sqrt{1+x^8}}+\sqrt{1+e^{3x}}-1$$

$$=\frac{2}{3}\ln\left(\sqrt{1+e^{3x}}-1\right)-x+C.$$

30 $$\int\frac{x^3}{4}dx=\frac{1}{4}\int\frac{dx^3}{\sqrt{1+x^8}}=\frac{1}{4}\int\frac{du}{\sqrt{1+u^2}}\left(u=x^4\right)$$

$$=\frac{1}{4}\ln\left(x^4+\sqrt{1+x^8}\right)+C.$$

31. $$\int\frac{dx}{x^6\sqrt{1+x^2}}=\int\frac{dx}{x^7\sqrt{1+x^{-2}}}=-\frac{1}{2}\int\frac{dx^{-2}}{x^4\sqrt{1+x^{-2}}}=-\frac{1}{2}\int\frac{u^2du}{\sqrt{1+u}}\left(u=\frac{1}{x^2}\right)$$

$$=-\frac{1}{2}\int\frac{(v-1)^2}{v^{1/2}}dv=-\frac{1}{2}\int\frac{v^2-2v+1}{v^{1/2}}dv(v=1+u)$$

$$=-\frac{1}{2}\int\left(v^{3/2}-2v^{1/2}+v^{-1/2}\right)dx$$

$$=-\frac{1}{2}\left(\frac{2}{5}v^{\frac{5}{2}}-2g_3^2v^{\frac{3}{2}}+2g^{\frac{1}{2}}\right)$$

$$=-\frac{1}{5}\left(1+\frac{1}{x^2}\right)^{\frac{5}{2}}+\frac{2}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}-\left(1+\frac{1}{x^2}\right)^{\frac{1}{2}}+C$$

$$=-\frac{\sqrt{1+x^2}}{5x^5}+\frac{\sqrt{1+x^2}^3}{3x^3}-\frac{\sqrt{1+x^2}}{x}+C.$$

32. $$\int\frac{e^{2x}}{\sqrt[3]{1+e^x}}dx=\int\frac{e^x}{\sqrt[3]{1+e^x}}de^x=\int\frac{u}{\sqrt[3]{1+u}}du\,(u=e^x)\,\left(\sqrt[3]{u+1}=v,u=v^3-1\right)$$

$$=\int\frac{u}{\sqrt[3]{1+u}}du=\int\frac{v^3-1}{v}3v^2dv=3\int\left(v^4-v\right)dv=3\left(\frac{v^5}{5}-\frac{v^2}{2}\right)+C$$

$$=\frac{3}{5}\left(e^x+1\right)^{5/3}-\frac{3}{2}\left(e^x+1\right)^{2/3}+C$$

33. $$\int\frac{dx}{\sqrt{3+x-x^2}}=\int\frac{dx}{\sqrt{3-\left(x-\frac{1}{2}\right)^2+\frac{1}{4}}}=\int\frac{d\left(x-\frac{1}{2}\right)}{\sqrt{\frac{13}{4}-\left(x-\frac{1}{2}\right)^2}}$$

$$=\arcsin\frac{x-\frac{1}{2}}{\frac{\sqrt{13}}{2}}+C=\arcsin\frac{2x-1}{\sqrt{13}}+C$$

34. $$\int\sqrt{7+x-x^2}dx=\int\sqrt{7-\left(x-\frac{1}{2}\right)^2+\frac{1}{4}}dx=\int\sqrt{\frac{29}{4}-\left(x-\frac{1}{2}\right)^2}d\left(x-\frac{1}{2}\right)$$

$$=\frac{1}{2}\left(x-\frac{1}{2}\right)\sqrt{\frac{29}{4}-\left(x-\frac{1}{2}\right)^2}+\frac{29}{8}\arcsin\frac{x-\frac{1}{2}}{4\frac{\sqrt{29}}{2}}+C$$

$$=\frac{2x-1}{4}\sqrt{7+x-x^2}+\frac{29}{8}\arcsin\frac{2x-1}{\sqrt{29}}+C.$$

(4)

$$35\cdot\int\frac{dx}{1+\sqrt{x-1}},1+\sqrt{x-1}=u,x=1+(u-1)^2,dx=2(u-1)du$$
$$\int\frac{dx}{1+\sqrt{x-1}},1+\sqrt{x-1}=u,x=1+(u-1)^2,dx=2(u-1)du$$
$$\int\frac{dx}{1+\sqrt{x-1}}==\int\frac{2(u-1)du}{u}=2(u-\ln u)+C=2(1+\sqrt{x-1})-\ln(1+\sqrt{x-1})\Big)+C$$
$$=2\sqrt{x-1}-\ln(1+\sqrt{x-1}))+C'$$

习题 3.2

1. $\int x\ln xdx=\frac{1}{2}\int\ln xdx^2=\frac{x^2}{2}\ln x-\frac{1}{2}\int x^2d\ln x$

$$=\frac{x^2}{2}\ln x-\frac{1}{2}\int x^2\ \mathrm{g}_x dx=\frac{x^2}{2}\ln x-\frac{1}{2}\int xdx=\frac{x^2}{2}\ln x-\frac{x^2}{4}+C$$

2. $\int x^2e^{ax}dx=\frac{1}{a}\int x^2de^{ax}=\frac{1}{a}x^2e^{ax}-\frac{1}{a}\int e^{ax}dx^2=\frac{1}{a}x^2e^{ax}-\frac{2}{a}\int xe^{ax}dx$

$$=\frac{1}{a}x^2e^{ax}-\frac{2}{a^2}\int xde^{ax}=\frac{1}{a}x^2e^{ax}-\frac{2x}{a^2}e^{ax}+\frac{2}{a^2}\int e^{ax}dx$$
$$=\frac{1}{a}x^2e^{ax}-\frac{2}{a^2}\int xde^{ax}=\frac{1}{a}x^2e^{ax}-\frac{2x}{a^2}e^{ax}+\frac{2}{a^3}e^{ax}+C$$
$$=e^{ax}\left(\frac{1}{a}x^2-\frac{2x}{a^2}+\frac{2}{a^3}\right)+C$$

3. $\int x\sin 2xdx=-\frac{1}{2}\int xd\cos 2x=-\frac{1}{2}x\cos 2x+\frac{1}{2}\int\cos 2xdx$

$$=-\frac{1}{2}x\cos 2x+\frac{1}{4}\sin 2x+C.$$

4. $\int\arcsin xdx=x\arcsin x-\int xd\arcsin x=x\arcsin x-\int\frac{xdx}{\sqrt{1-x^2}}$

$$=x\arcsin x+\frac{1}{2}\int\frac{d\left(1-x^2\right)}{\sqrt{1-x^2}}=x\arcsin x+\sqrt{1-x^2}+C \tag{5}$$

5. $\int\arctan xdx=x\arctan x-\int xd\arctan x=x\arctan x-\int\frac{xdx}{1+x^2}$

$$=x\arctan x-\frac{1}{2}\int\frac{d\left(1+x^2\right)}{1+x^2}=x\arctan x-\frac{1}{2}\ln\left(1+x^2\right)+C$$

$$6I=\int e^{2x}\cos 3xdx=\frac{1}{2}\int\cos 3xde^{2x}=\frac{1}{2}e^{2x}\cos 3x-\frac{1}{2}\int e^{2x}d\cos 3x$$
$$=\frac{1}{2}e^{2x}\cos 3x+\frac{3}{2}\int e^{2x}\sin 3xdx=\frac{1}{2}e^{2x}\cos 3x+\frac{3}{4}\int\sin 3xde^{2x}$$
$$=\frac{1}{2}e^{2x}\cos 3x+\frac{3}{4}\left(e^{2x}\sin 3x-3\int e^{2x}\cos 3xdx\right)$$
$$=\frac{1}{2}e^{2x}\cos 3x+\frac{3}{4}e^{2x}\sin 3x-\frac{9}{4}I$$
$$I=\frac{4}{13}\left(\frac{1}{2}\cos 3x+\frac{3}{4}\sin 3x\right)e^{2x}+C=\frac{1}{13}(2\cos 3x+3\sin 3x)e^{2x}+C$$
$$7I=\int\frac{\sin 3x}{e^x}dx=-\int\sin 3xde^{-x}=-e^{-x}\sin 3x+3\int e^{-x}\cos 3xdx$$
$$=-e^{-x}\sin 3x-3\int\cos 3xde^{-x}=-e^{-x}\sin 3x-3\left(e^{-x}\cos 3x+3\int e^{-x}\sin 3xdx\right)$$

$= x(\arccos x)^2 - 2\sqrt{1-x^2}\arccos x - 2x + C.$

13. $\int \frac{x\arccos x dx}{(1-x^2)^2} = \frac{1}{2}\int \arccos x d\frac{1}{1-x^2}$

$= \frac{\arccos x}{2(1-x^2)} + \frac{1}{2}\int \frac{dx}{(1-x^2)\sqrt{1-x^2}}$

$= \frac{\arccos x}{2(1-x^2)} + \frac{1}{2}\frac{x}{\sqrt{1-x^2}} + C.$

14. $\int \arctan\sqrt{x}dx = x\arctan\sqrt{x} - \int \frac{xdx}{2(1+x)\sqrt{x}}$

$= x\arctan\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}dx}{1+x} \cdot \sqrt{x} = u, x = u^2, dx = 2udu$

$\int \frac{\sqrt{x}dx}{1+x} = \int \frac{u2udu}{1+u^2} = 2(u - \arctan u) + C$

$\int \arctan\sqrt{x}dx = x\arctan\sqrt{x} - \frac{1}{2}g2(\sqrt{x} - \arctan\sqrt{x}) + C$

$= x\arctan\sqrt{x} - (\sqrt{x} - \arctan\sqrt{x}) + C$

$= (x+1)\arctan\sqrt{x} - \sqrt{x} + C$

15. $\int \frac{\arcsin x}{x^2}dx = -\int \arcsin x d\left(\frac{1}{x}\right) = -\frac{\arcsin x}{x} + \int \frac{dx}{x\sqrt{1-x^2}}$

$= -\frac{\arcsin x}{x} + \int \frac{dx}{x^2\sqrt{1/x^2-1}} (x > 0)$

$= -\frac{\arcsin x}{x} - \int \frac{d(1/x)}{\sqrt{1/x^2-1}} = -\frac{\arcsin x}{x} - \ln\left|1/x + \sqrt{1/x^2-1}\right| + C$

$= -\frac{\arcsin x}{x} + \ln\left(1 - \sqrt{1-x^2}\right) - \ln x + C$

$= -\frac{\arcsin x}{x} + \ln\left(1 - \sqrt{1-x^2}\right) - \ln|x| + C(x \neq 0)$ (原函数为偶函数).

16. $\int x^3(\ln x)^2 dx = \frac{1}{4}\int (\ln x)^2 dx^4 = \frac{x^4(\ln x)^2}{4} - \frac{1}{4}\int \frac{x^4 \text{ g}\ln x dx}{x}$

$= \frac{x^4(\ln x)^2}{4} - \frac{1}{2}\int x^3 \ln x dx = \frac{x^4(\ln x)^2}{4} - \frac{1}{8}\int \ln x dx^4$

$= \frac{x^4(\ln x)^2}{4} - \frac{x^4}{8}\ln x + \frac{1}{2}\int x^3 dx = \frac{x^4(\ln x)^2}{4} - \frac{x^4}{8}\ln x + \frac{1}{8}x^4 + C$

17. $\int \frac{x\arctan x dx}{(1+x^2)^{5/2}} = \frac{1}{2}\int \frac{\arctan x d\left(1+x^2\right)}{(1+x^2)^{5/2}} = \frac{1}{2}g - \frac{2}{3}\Big)\int \arctan x d\left(1+x^2\right)^{-3/2}$

(6)

$$= -\frac{\arctan x}{3\left(1+x^2\right)^{3/2}} + \frac{1}{3}\int \frac{dx}{\left(1+x^2\right)^{5/2}} x = \tan u, u \in (-\pi/2, \pi/2) \cdot dx = \sec^2 u du$$

$$\int \frac{dx}{\left(1+x^2\right)^{5/2}} = \int \cos^3 u du = \int \left(1-\sin^2 u\right) d\sin u =$$

$$= \sin u - \frac{1}{3}\sin^3 u + C = \frac{x}{\sqrt{1+x^2}} - \frac{1}{3}\left(\frac{x}{\sqrt{1+x^2}}\right)^3 + C$$

$$\int \frac{x \arctan x dx}{\left(1+x^2\right)^{5/2}} = -\frac{\arctan x}{3\left(1+x^2\right)^{3/2}} + \frac{1}{3}\left(\frac{x}{\sqrt{1+x^2}} - \frac{1}{3}\left(\frac{x}{\sqrt{1+x^2}}\right)^3\right) + C$$

$$= -\frac{\arctan x}{3\left(1+x^2\right)^{3/2}} + \frac{1}{3}\frac{x}{\sqrt{1+x^2}} - \frac{1}{9}\frac{x^3}{\left(1+x^2\right)^{3/2}} + C$$

18. $$\int x \ln\left(x+\sqrt{1+x^2}\right) dx = \frac{1}{2}\int \ln\left(x+\sqrt{1+x^2}\right) dx^2 \tag{7}$$

$$= \frac{1}{2}x^2 \ln\left(x+\sqrt{1+x^2}\right) - \frac{1}{2}\int \frac{x^2 dx}{\sqrt{1+x^2}}$$

$$= \frac{1}{2}x^2 \ln\left(x+\sqrt{1+x^2}\right) - \frac{1}{2}\int \frac{\left(x^2+1\right)-1 dx}{\sqrt{1+x^2}}$$

$$= \frac{1}{2}x^2 \ln\left(x+\sqrt{1+x^2}\right) - \frac{1}{2}\int \sqrt{1+x^2} dx + \frac{1}{2}\int \frac{dx}{\sqrt{1+x^2}}$$

$$= \frac{1}{2}x^2 \ln\left(x+\sqrt{1+x^2}\right) - \frac{1}{2}\left(\frac{x\sqrt{1+x^2}}{2} + \frac{\ln\left(x+\sqrt{1+x^2}\right)}{2}\right) + \frac{1}{2}\ln\left(x+\sqrt{1+x^2}\right) + C$$

$$= \frac{1}{2}x^2 \ln\left(x+\sqrt{1+x^2}\right) - \frac{1}{4}x\sqrt{1+x^2} + \frac{1}{4}\ln\left(x+\sqrt{1+x^2}\right) + C.$$