偏微分方程数值解　序言

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今日计划

课程大纲和设计

什么是微分方程数值解？

内容预览

参考书目

- 李治平: 偏微分方程数值解讲义，北京大学出版社。
- Morton, Mayers: Numerical Solution of Partial Differential Equations, Cambridge University Press, 中译本, 人民邮电出版社。
- 李荣华、冯果忱：微分方程数值解法（第三版），高等教育出版社。
- 胡祖织、雷功炎: 偏微分方程初值问题差分方法，北京大学出版社。
- 应隆安: 有限元方法讲义，北京大学出版社。
- 汤怀民、胡健伟: 微分方程数值方法, 南开大学出版社。
- Randall J. LeVeque, Finite Difference Methods for Ordinary and Partial Differential Equations, Society for Industrial and Applied Mathematics (SIAM), Philadelphia.
- Claes Johnson, Numerical solution of partial differential equations by the finite element method, Cambridge University Press.

注：**只需要买教材 1**。

教学内容

本学期教学大纲大致包括:

微分方程基础和椭圆型方程的差分方法、抛物型方程的有限差分方法、双曲型方程的有限差分方法、线性发展型方程有限差分方法的一般理论和拓展选讲、椭圆边值问题的变分形式、椭圆边值问题的有限元方法、椭圆边值问题有限元解的误差估计、微分方程数值解的其他专题选讲……等等

讲义、作业、期中试卷都会定期更新在

课程主页:

http://bicmr.pku.edu.cn/ zhennan/teaching/NP22/

办公室地址

78406-02, 镜春园78号院。

Office Hour:

周三上午10-11点半。

评分方法:

- A：两周一次作业，单周布置作业（除了第7周，第15周之外），于次周的周日（23：59）前在教学网提交。共6次作业。所有作业总计100分，占学期总分的30%；
- B (B1+B2): 第7周和第15周，我们会布置上机作业，每次作业满分100分，分别占学期总分的10%，共20%。
- C：期末考试，笔试，满分为100分，占学期总分的50%。
- 也就是说，你的学期总分=A*0.3+(B1+B2)*0.1+C*0.5。

注意：你的总分是卷面分数，我们有可能会因为优秀率的限制在不改变名次的条件下进行整体调分。

其他注意事项

作业要求:

不接受晚交的作业和term project。老师鼓励同学们之间讨论作业题目，但是每个同学必须自己独立写作业。作业可以手写，也可以用电脑写（word, latex等方式）再打印出来。作业的收发由助教老师负责

Office hour 和助教职责

本学期老师和助教每周都会设置office hour答疑。同学也可以通过电子邮件联系我们答疑。请大家注意，我们的微信群是主要供同学们相互交流讨论的，老师和助教不在微信群里答疑。

助教

- 韩如冰，hanrubing@pku.edu.cn。
  Office Hour：周一上午9-11点，21楼309。

科学计算

- 科学计算是指利用计算机再现、预测和发现客观世界运动规律和演化特征的全过程。科学计算为解决科学和工程中的数学问题利用计算机进行的数值计算。其核心和基础是计算数学，或数值分析。
- 科学计算是20世纪重要科学技术进步之一,已与**理论研究**和**实验研究**相并列成为科学研究的**第三种方法**。
- 在迅猛发展的高性能计算技术推动下,计算科学将是21世纪确保国家核心竞争能力的战略技术之一。

微分方程数值方法

有限差分法，有限元法，有限体积法，谱方法，边界积分法，无网络方法，随机模拟法，快速算法……

这是一门什么样的课？

Arieh Iserles 在他的著作《A First Course in the Numerical Analysis of Differential Equations》中写了两版序言，分享了他对于数值微分方程的教学中很多有意义的反思。

《Preface to the first edition》

“Books – so we are often told – should be born out of a sense of mission, a wish to share knowledge, experience and ideas, a penchant for beauty. This book has been born out of **a sense of frustration**.”

“Examining this extensive period of trial and (frequent) error, **two main conclusions** come to mind and both have guided my choice of material and presentation in this volume.”

科学计算对于数学人来说是什么？

"Firstly, mathematicians are different from other varieties of *homo sapiens*."

- **Scientists and engineers** require it as a means to an end, a tool to investigate the subject matter that *really* interests them.
- **Computer scientists** adopt a different, more algorithmic, attitude. Their heart goes after the clever algorithm and its interaction with computer architecture.
- **Mathematicians**, though, follow a different mode of reasoning. Numerical analysis does not fit easily into the familiar sequence of axiom, theorem, proof, corollary...
  ...namely to present the material in a *bona fide* mathematical manner, occasionally veering toward issues of applications and algorithmics but never abandoning honesty and rigour.

Secondly, 内容很多，讲什么？

一些刻板印象

《Preface to the second edition》

"I have found myself, as I often do, thanking my good fortune for working in this particular corner of the mathematical garden, the numerical analysis of differential equations, and striving in a small way to communicate its oft-unappreciated beauty."

Numerical analysis is often considered neither beautiful nor, indeed, profound.

- **Pure mathematics** is beautiful if your heart goes after the joy of abstraction;
- **Applied mathematics** is beautiful if you are excited by mathematics as a means to explain the mystery of the world around us.
- But **numerical analysis**? Surely, we compute only when everything else fails...throw to a computer... produce boring numbers by boring calculations. **This, I believe, is nonsense.**

数值分析作为一个数学分支

"**A mathematical problem does not cease being mathematical just because we have discretized it.**"

"once we wish to answer the big mathematical question underlying our discourse, how well does the finite-dimensional model approximate the original equation, we are compelled to consider not one finite-dimensional system but **an infinite progression of such systems, of increasing (and unbounded) dimension**."

"Actually, it is almost inevitable that a discretized mathematical problem is, as a mathematical problem, **more difficult and more demanding of our mathematical ingenuity**."

"My stronger claim, illustrated in a small way by the material of this book, is that numerical analysis is perhaps the most eclectic and demanding client of the entire width and breadth of mathematics."

内容预览

- 第一章：微分方程基础和椭圆型方程的差分方法、
- 第二章：抛物型方程的有限差分方法、
- 第三章：双曲型方程的有限差分方法、
- 第四章：线性发展型方程有限差分方法的一般理论和拓展选讲、
- 第五章：椭圆边值问题的变分形式、
- 第六章：椭圆边值问题的有限元方法、
- 第七章：椭圆边值问题有限元解的误差估计、
- 第八章：微分方程数值解的其他专题选讲